ELI-NP at Magurele - "Pulse and Impulse of ELI"

1)"**Polaritonic pulse** and coherent X- and gamma rays from Compton (Thomson) backscattering" (MApostol&MGanciu), J. Appl. Phys. **109** 013307 (2011) (1-6)

2)"Dynamics of **electron–positron pairs** in a vacuum polarized by an external radiation field" (MA), Journal of Modern Optics, **58** 611 (2011)

3)"**Classical interaction** of the electromagnetic radiation with two-level polarizable matter" (MA), Optik **123** 193 (2012)

4)"**Coherent polarization** driven by external electromagnetic fields" (MA&MG), Physics Letters **A374** 4848 (2010)

5)"Coupling of **(ultra-) relativistic atomic nuclei** with photons" (MA&MG), AIP Advances **3** 112133 (2013)

6)"Propagation of **electromagnetic pulses** through the surface of dispersive bodies" (MA), Roum J. Phys. **58** 1298 (2013)

7)"**Giant dipole oscillations** and ionization of heavy atoms by intense electromagnetic pulses" (MA)...

What we should know firstly about lasers:

Intensity $I \simeq \mathcal{E}/d^2\tau$;

1)Threshold (1990) : $I = 10^{14} - 10^{15} w/cm^2$(CPA); $E \simeq \sqrt{I/c} \simeq 10^6 (esu)(3 \times 10^4 V/m)$ - atomic fields - ionization, HO harmonics

2) Current: $I = 10^{18} - 10^{19} w/cm^2$, $E \simeq 10^8$ - relativistic, non-linear

3) Few claims: $10^{21} - 10^{22} w/cm^2$ (ultrahigh), $E \simeq 10^9$ - multi-photon, radiation reaction, q vacuum, particle creation

Schwinger limit $E \simeq 10^{13}$

Power $P \simeq \mathcal{E}/\tau \; (Id^2)$

Current: $\tau \simeq x \cdot 10^{-15}s$, $d \simeq 3x \cdot 10^{-5}cm$

The petawatt: $10^{21} \times (9x^2)10^{-10} = 10^{15}w \; (x = 50)$!

Wavelength $\varepsilon = 1eV$, $\omega = 10^{15}s^{-1}$, $\lambda = 10^{-5}cm$ (tens of λ in a pulse)

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Giant dipole oscillations and ionization of heavy atoms by intense electromagnetic fields

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Thomas-Fermi model (1927)

$$\hbar^2 k_F^2/2m - e\varphi = 0\ ,\ k_F = (2me/\hbar^2)^{1/2}\varphi^{1/2}$$

Poisson's equation

$$\Delta\varphi = -4\pi Ze\delta(\mathbf{r}) + 4\pi ne\ ,\ n = k_F^3/3\pi^2 = (1/3\pi^2)(2me/\hbar^2)^{3/2}\varphi^{3/2}$$

$$\Delta\varphi = -4\pi Ze\delta(\mathbf{r}) + \frac{4e}{3\pi}(2me/\hbar^2)^{3/2}\varphi^{3/2}$$

Solution: Numerical

Binding energy $E \simeq -20.8 Z^{7/3}$eV, exact for $Z \to \infty$ (Lieb&Simon, 1977)

Empirical: $E \simeq -16 Z^{7/3}$eV ($Z > 20$)

Corrections: (asymptotic) series in $Z^{-1/3}$; (convergence?)

Another drawback: no-binding theorem (Teller, 1962).

Linearized version (with L C Cune; binding)

$k_F^2 \to 2\overline{k}_F k_F$, $k_F^3 \to 3\overline{k}_F^2 k_F$, etc (quasi-classical approximation)

Alternatively,

$$\frac{\hbar^2}{m} k_F \delta k_F - e\delta\varphi = 0 \ , \ k_F = (me/\hbar^2 \overline{k}_F)\varphi$$

$$n = (me\overline{k}_F/\pi^2\hbar^2)\varphi = (q^2/4\pi e)\varphi$$

Thomas-Fermi screening wavevector q

$$q^2 = 4me^2\overline{k}_F/\pi\hbar^2 = \frac{4\overline{k}_F}{\pi a_H} \ , \ a_H = \frac{\hbar^2}{me^2} = 0.53\text{Å}$$

$$\Delta\varphi = -4\pi Ze\delta(\mathbf{r}) + q^2\varphi \ , \ \varphi = \frac{Ze}{r}e^{-qr}$$

Energy

$$E_{kin} = V\hbar^2 k_F^5/10\pi^2 m \to (\hbar^2 \overline{k}_F^4/2\pi^2 m) \int d\mathbf{r} \cdot k_F = \frac{\pi e a_H^3}{128} q^6 \int d\mathbf{r} \cdot \varphi$$

$$E_{kin} = \frac{\pi^2 a_H^3}{32} Z e^2 q^4$$

$$E_{pot} = \int d\mathbf{r}(\rho_e \varphi - \tfrac{1}{2}\rho_e \varphi_e) = \tfrac{1}{2}\int d\mathbf{r}(\rho_e \varphi + \rho_e \varphi_c) =$$

$$= -\tfrac{e}{2}\int d\mathbf{r} n(\varphi + \varphi_c) = -\tfrac{q^2}{8\pi}\int d\mathbf{r}(\varphi^2 + \varphi\varphi_c)$$

$\rho_e = -en,\ \varphi_e = \varphi - \varphi_c,\ \varphi_c = Ze/r$

$$E_{pot} = -\frac{3}{4} Z^2 e^2 q$$

Total energy

$$E = E_{kin} + E_{pot} = \frac{\pi^2 a_H^3}{32} Z e^2 q^4 - \frac{3}{4} Z^2 e^2 q$$

Minimum value

$$E = -\frac{9 \cdot 6^{1/3}}{16\pi^{2/3}} Z^{7/3} \frac{e^2}{a_H} = -0.42 Z^{7/3} \frac{e^2}{a_H} = -11.4 Z^{7/3} eV$$

for

$$q = (6/\pi^2)^{1/3} \frac{Z^{1/3}}{a_H} = 0.85 \frac{Z^{1/3}}{a_H}$$

(atomic unit for energy $e^2/a_H \simeq 27.2$eV)

Electron distribution:

r^2n, maximum for $R \sim 1/q \sim Z^{-1/3}a_H$, the "radius" of the electronic charge

Quasi-classical description: $a_H/Z \ll R \sim a_H/Z^{1/3} \ll a_H$

($a_H = \hbar^2/me^2 = 0.53\text{Å}$)

Quantum correction

$$-\frac{e}{v}\int_v d\mathbf{r}\cdot\varphi = -3Ze^2q\cdot\frac{1}{x^3}(1-e^{-x}-xe^{-x})\ ,\ v=4\pi R^3/3\ ,\ x=qR$$

Multiply by $\int_v d\mathbf{r}\cdot n$

$$\Delta E = \frac{16}{3}E\cdot\frac{1}{x^3}(1-e^{-x}-xe^{-x})^2$$

Minimize with respect to R (maximum 0.073 for $x \simeq 0.75$),

$$\Delta E = 0.39E = -4.44Z^{7/3}\text{eV}$$

Total energy

$$E = -11.4Z^{7/3}\text{eV} - 4.44Z^{7/3}\text{eV} = -15.84Z^{7/3}\text{eV}$$

Compare with empirical $E \simeq -16Z^{7/3}$eV

Giant dipole oscillations

$Rq = const$; $u = -(1/q^2)\delta q$

$$\delta E = \delta E_{kin} = \frac{27}{4\pi^2}\frac{Z^3 e^2}{a_H^3}u^2$$

frequency ω_0, $\delta E = \frac{1}{2}M\omega_0^2 u^2$, $M = Zm$

$$\omega_0 = \left(\frac{27}{2\pi^2}\right)^{1/2}\frac{Ze}{\sqrt{ma_H^3}} \simeq 4.5Z \times 10^{16}s^{-1}$$

energy $\hbar\omega_0 \simeq 28ZeV$, moderate X-rays

Wavelength $\lambda_0 = 2\pi c/\omega_0 \simeq \frac{4.2}{Z} \times 10^{-6}cm$ much larger than the dimension of the atom

Damping force: $F_d = 2Q^2 \ddot{v}/3c^3$, $Q = -Ze$, $v = \dot{u}$

$\ddot{v} = \omega_0^2 v$, $F_d = \frac{2Z^2e^2}{3c^3}\omega_0^2 v = M\gamma\dot{u}$, $M = Zm$

$$\gamma = \frac{2Ze^2}{3mc^3}\omega_0^2$$

Note: $\gamma \ll \omega_0$, since

$$\frac{2}{3}Z\frac{e^2}{mc^2} \ll \frac{c}{\omega_0}\ , \ \frac{4\pi}{3}Zr_0 \ll \lambda_0$$

Class em radius $e^2/mc^2 = r_0 \simeq 2.8 \times 10^{-13} cm$

Quality ratio (natural breadth of the spectroscopic line)

$$\frac{\gamma}{\omega_0} = \frac{4\pi Z r_0}{3\lambda_0} \simeq 2.8 Z^2 \times 10^{-7}$$

Equation of motion

$$M\ddot{u} + M\omega_0^2 u + M\gamma\dot{u} = QE\cos\omega t\ ,\ m\ddot{u} + m\omega_0^2 u + m\gamma\dot{u} = -eE\cos\omega t$$

Solution

$$u = a\cos\omega t + b\sin\omega t$$

$$a = \frac{eE}{m}\frac{\omega^2 - \omega_0^2}{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}\ ,\ b = -\frac{eE}{m}\frac{\omega\gamma}{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}$$

Energy conservation, power loss

$$P = m\gamma\overline{\dot{u}^2} = \frac{e^2E^2}{2m}\frac{\gamma\omega^2}{(\omega^2 - \omega_0^2)^2 + \omega^2\gamma^2}$$

Resonance

$$P_{res} = \frac{e^2 E^2}{2m\gamma}$$

Moderate fields $E = 1/300 esu$ $(100V/m)$:

$$P_{res} = \frac{1}{Z^3} \times 10^{-7} erg/s$$

Transition rate

$$R = P_{res}/\hbar\omega_0 \simeq \frac{2}{Z^4} \times 10^3 s^{-1}$$

(Radiation)

Partial ionization

Linear Th-F model:

$$\omega_0^{'} = (\delta Z/Z)\omega_0$$

$$\gamma^{'} = (2e^2 \delta Z/3mc^3)\omega_0^{'2}$$

(UV)

Anharmonicities and ionization

Amplitude (at resonance) (compare with $a_H = 0.53$Å)

$$|b_0| = \frac{eE}{m\omega_0\gamma} = \frac{8}{Z^4} \times 10^{-10} E\, cm$$

Small, harmonic approximation; higher-order corrections

Potential energy ($q = 0.85 Z^{1/3}/a_H$)

$$U = \frac{1}{2} M\omega_0^2 u^2 \left(1 - \frac{8}{3} qu + \frac{31}{6} q^2 u^2 + ...\right)$$

Combined frequencies, harmonics, freq shifts

Successive approximations, SCHOA

Ionization $((q+\delta q)(R+u)=1$ $(\delta q=-q^2u/(1+qu)$ $)$

$\Rightarrow U_\infty = 9Z^2e^2q/16$ (with $q=0.85Z^{1/3}/a_H$)$=-E$

Complete ionization (hyper-ionization, dissociation) $|b_0|>a_H$

Critical field $E>7Z^4$; high field, for (X-ray) frequency ω_0

Partial ionization δZ $(1\ll\delta Z<Z)$: $E>7(\delta Z)^4$

Ionization potential: $\hbar\omega_0=30eV$ too large ($6eV$)

Quasi-classical approximation

$\dot{O} = \frac{i}{\hbar}[H, O]$, $\dot{O}_{mn} = \frac{i}{\hbar}(E_m - E_n)O_{mn} = i\omega_{mn}O_{mn}$

Densely distributed states: $\omega_{mn} \simeq -s(\partial E_m/\partial m)_m = -\omega_s$

Matrix elements $O_{mn} = O_{m,m+s} \simeq O_s$ (temporal Fourier transform)

$\dot{O} = -i\omega_s O + (\partial O^{cl}/\partial t)_{cl;h}$; $O = O^{(1)} + iO^{(2)}$, $\dot{O}^{(1)} = \omega_s O^{(2)} + (\partial O^{cl}/\partial t)_{cl;h}$, $\dot{O}^{(2)} = -\omega_s O^{(1)}$, $\ddot{O}^{(1)} + \omega_s^2 O^{(1)} = (\partial/\partial t)(\partial O^{cl}/\partial t)_{cl;h}$

$$\ddot{O} + \omega_s^2 O = (\partial/\partial t)(\partial O^{cl}/\partial t)_{cl;h} = f$$

Classical harmonic oscillator; eigenfrequency ω_s is the quantum transition frequency ω_{mn} (wavepacket, lifetime γ^{-1})

QClass Ionization: "Peripheral" electrons in heavy atoms are quasi-class

External field $h = eEu\cos\omega t$, force $f = -eE\cos\omega t$

$$\ddot{u} + \omega_0^2 u + \gamma\dot{u} = -\frac{e}{m}E\cos\omega t$$

ω_0 $(= \omega_s)$ - ionization frequency (exc states)

Amplitude $|b_0| = eE/m\omega_0\gamma$; $\gamma = 2r_0\omega_0^2/3c = 6\times10^{-24}\omega_0^2$

$\omega_0 = 10^{16}s^{-1}$ $(\hbar\omega_0 = 6eV)$ $\Rightarrow |b_0| = 10^{-7}E\,cm$

Critical field $E > 5\times10^{-2} statvolt/cm$ $(\simeq 1500V/m)$

Higher fields: multiple-quanta transitions; Qclass: larger displacements, anharmonicities $(\omega_0^2 = (1/m)(\partial^2 U/\partial u^2)_0)$