Climbing scalars and implications for Cosmology

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- E. D, N. Kitazawa, A.Sagnotti, *P.L.* ***B*** **694** (2010) 80 [arXiv:1009.0874 [hep-th]].
- E. D, N. Kitazawa, S. Patil, A.Sagnotti, *JCAP* **1205** (2012) 012 [arXiv:1202.6630 [hep-th]]
- E. D, N. Kitazawa, S. Patil, A.Sagnotti, in progress
- C. Condeescu, E.D., in progress

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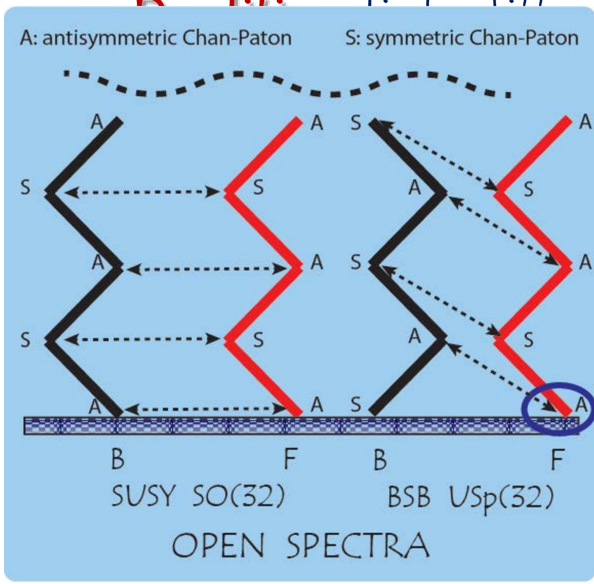
Outline

- Brane SUSY breaking
- A climbing scalar in D dimensions
- Climbing with a SUSY axion (KKLT)
- Climbing and inflation, power spectrum
- Kasner approach: higher-derivative corrections, models with no big-bang
- Outlook

Brane SUSY Breaking

(Sugimoto, 1999)
(Antoniadis, E.D, Sagnotti, 1999)
(Aldazabal, Uranga, 1999)
(Angelantonj, 1999)



...ent strings

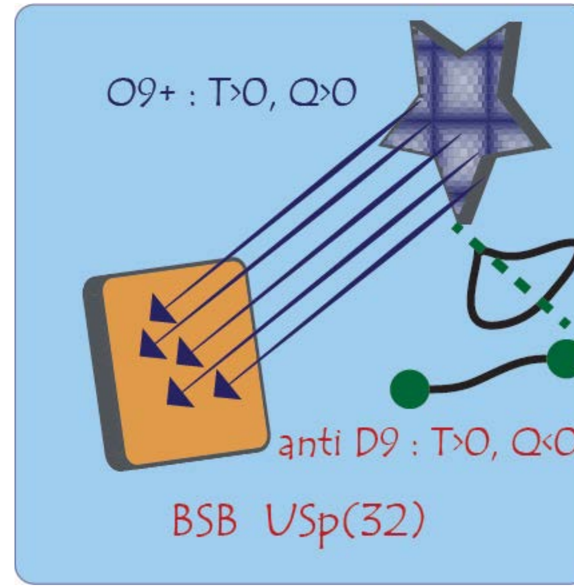
...osed and open strings

Tree – level BSB

$O9_-$ (T < 0, Q < 0) → SO(32)

+ $O9_+$ (T > 0, Q > 0) → USp(32)

- SUSY broken at string scale in open sector, exact in closed
- Stable vacuum
- Goldstino in open sector



BSB: Tension unbalance → exponential potential

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\ \{ e^{-2\phi}\left(-R + 4(\partial\phi)^2\right) - T\,e^{-\phi} + \ldots \}$$

- **Flat space :** runaway behavior
- **String-scale breaking :** early-Universe Cosmology ?

A climbing scalar in d dim's

- Consider the action for gravity and a scalar ϕ :

$$S = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \ldots \right]$$

- Look for **cosmological solutions** of the type

$$ds^2 = -\, e^{2\mathcal{B}(t)}\, dt^2 + e^{2A(t)}\, d\mathbf{x} \cdot d\mathbf{x}$$

(Halliwell, 1987)
....
(E.D,Mourad, 2000)
(Russo, 2004)
....

- Make the convenient **gauge choice** $V(\phi)\, e^{2\mathcal{B}} = M^2$

- Let :

$$\beta = \sqrt{\frac{d-1}{d-2}}\,, \qquad \tau = M\,\beta\, t\,, \qquad \varphi = \frac{\beta\,\phi}{\sqrt{2}}\,, \qquad \mathcal{A} = (d-1)\, A$$

- In expanding phase :

$$\ddot{\varphi} + \dot{\varphi}\sqrt{1 + \dot{\varphi}^2} + \left(1 + \dot{\varphi}^2\right) \frac{1}{2V} \frac{\partial V}{\partial \varphi} = 0$$

- OUR CASE :

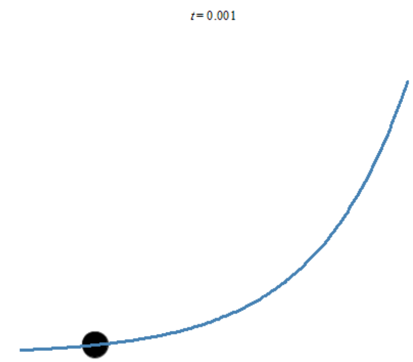
$$V = \exp(2\,\gamma\,\varphi) \longrightarrow \frac{1}{2\,V} \frac{\partial V}{\partial\,\varphi} = \gamma$$

A climbing scalar in d dim's

- γ < 1 ?   Both signs of speed

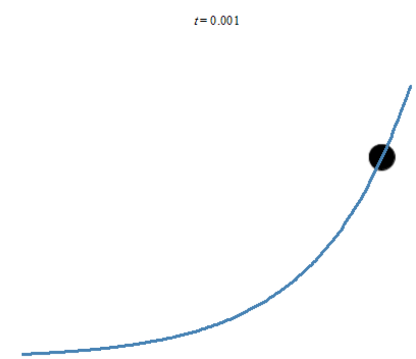
a. "Climbing" solution (φ climbs, then descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



b. "Descending" solution (φ only descends):

$$\dot{\varphi} = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



NOTE: only φ_o. Early speed → singularity time!

Limiting τ- speed (LM attractor): $v_l = -\frac{\gamma}{\sqrt{1-\gamma^2}}$

γ → 1 : LM attractor & descending solution disappear

- γ ≥ 1 ? Climbing! E.g. for γ=1 : $\dot{\varphi} = \frac{1}{2\tau} - \frac{\tau}{2}$

CLIMBING: in ALL asymptotically exponential potentials with γ≥1 !

String Realizations

NOTE: a. Two - derivative couplings: α' corrections ? *(C.Condeescu, E.D, in progress)*
b. [BUT: climbing → weak string coupling]

Dimensional reduction of (critical) 10-dimensional low-energy EFT:

$$S_D = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\,\{ e^{-2\phi} \left(-R + 4(\partial\phi)^2\right) - T\,e^{-\phi} + \ldots\}$$

$$ds^2 = e^{-\frac{(10-d)}{(d-2)}\sigma} g_{\mu\nu}\,dx^\mu\,dx^\nu + e^{\sigma}\,\delta_{ij}\,dx^i\,dx^j$$

$$S_d = \frac{1}{2\kappa_d^2} \int d^dx\sqrt{-g}\left\{ -R - \frac{1}{2}(\partial\phi)^2 - \frac{2(10-d)}{(d-2)}(\partial\sigma)^2 - T\,e^{\frac{3}{2}\phi - \frac{(10-d)}{(d-2)}\sigma} + \ldots\right\}$$

- Two scalar combinations (Φ_s and Φ_t). Focus on Φ_t :

$$S_d = \frac{1}{2\kappa_d^2} \int d^dx\sqrt{-g}\left\{ -R - \frac{1}{2}(\partial\Phi_s)^2 - \frac{1}{2}(\partial\Phi_t)^2 - T\,e^{\Delta\,\Phi_t}\right\}$$

$$\Delta = \sqrt{\frac{2(d-1)}{(d-2)}} \quad \Rightarrow \quad \gamma = 1 \quad \forall d < 10!$$

Climbing with a SUSY Axion

(Kachru, Kallosh, Linde, Trivedi, 2003)

- No-scale reduction + 10D tadpole → KKLT uplift

(Cremmer, Ferrara, Kounnas, Nanopoulos, 1983)
(Witten, 1985)

$$T = e^{-\frac{\Phi_t}{\sqrt{3}}} + i\,\frac{\theta}{\sqrt{3}}$$

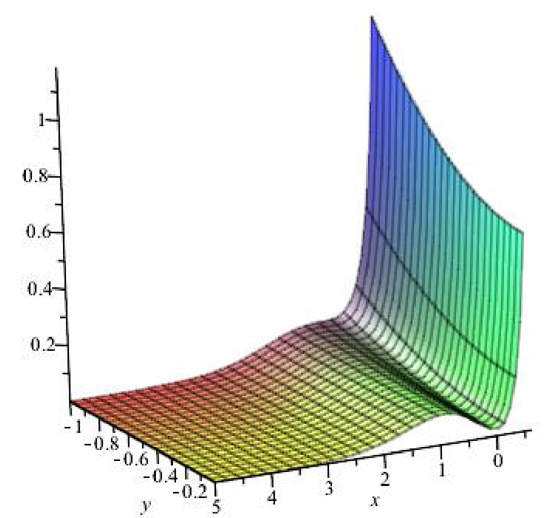
$$S_4 = \frac{1}{2\kappa_4^2}\int d^4x\,\sqrt{-g}\,\left\{R - \frac{1}{2}(\partial\Phi_t)^2 - \frac{1}{2}\,e^{\frac{2}{\sqrt{3}}\Phi_t}(\partial\theta)^2 - V(\Phi_t,\theta) + \cdots\right\}$$

$$V(\Phi_t,\theta) = \frac{c}{(T+\bar{T})^3} + V_{(non\ pert.)}$$

$$\Phi_t = \frac{2}{\sqrt{3}}\,x\ ,\quad \theta = \frac{2}{\sqrt{3}}\,y$$

$$\frac{d^2x}{d\tau^2} + \frac{dx}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} + \frac{1}{2V}\frac{\partial V}{\partial x}\left[1+\left(\frac{dx}{d\tau}\right)^2\right] + \frac{1}{2V}\frac{\partial V}{\partial y}\frac{dx}{d\tau}\frac{dy}{d\tau} - \frac{2}{3}\,e^{\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2 = 0\ ,$$

$$\frac{d^2y}{d\tau^2} + \frac{dy}{d\tau}\sqrt{1+\left(\frac{dx}{d\tau}\right)^2 + e^{\frac{4x}{3}}\left(\frac{dy}{d\tau}\right)^2} + \left(\frac{1}{2V}\frac{\partial V}{\partial x} + \frac{4}{3}\right)\frac{dx}{d\tau}\frac{dy}{d\tau} + \frac{1}{2V}\frac{\partial V}{\partial y}\left[e^{-\frac{4x}{3}} + \left(\frac{dy}{d\tau}\right)^2\right] = 0$$

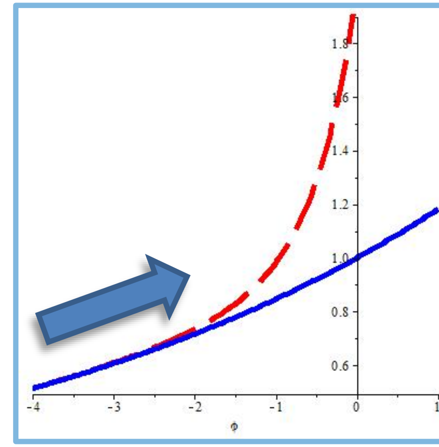


AXION INITIALLY "FROZEN" → CLIMBING!

Climbing and Inflation

a. "Hard" exponential of Brane SUSY Breaking

b. "Soft" exponential ($\gamma < 1/\sqrt{3}$):



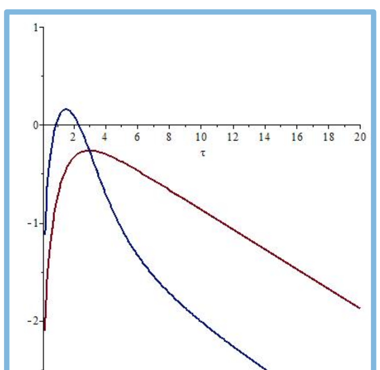
Would need : $\gamma \approx \frac{1}{12}$

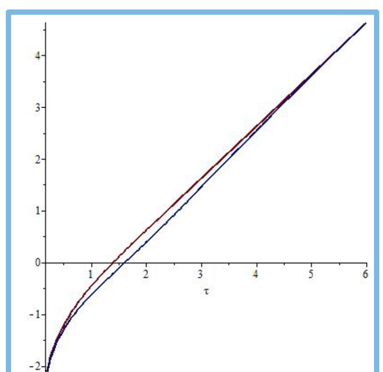
$$V(\phi) = \overline{M}^{\,4} \left(e^{2\varphi} + e^{2\gamma\varphi}\right)$$

Non-BPS D3 brane gives $\gamma = 1/2$
[+ stabilization of Φ_s]

(Sen , 1998)
(E.D J.Mourad, A.Sagnotti 2001)

- BSB "Hard exponential" → makes initial **climbing** phase **inevitable**
- "Soft exponential" → drives **inflation** during subsequent descent





φ_o : "hardness" of kick !

Mukhanov – Sasaki Equation

Schroedinger-like equation for scalar (or tensor) fluctuations :

$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\mathbf{x}\cdot d\mathbf{x}\right)$$

$$Scalar : z(\eta) = a^2(\eta)\,\frac{\phi_0'(\eta)}{a'(\eta)}$$

$$Tensor : z(\eta) = a$$

$$W_s = \frac{1}{z}\,\frac{d^2 z}{d\eta^2}$$

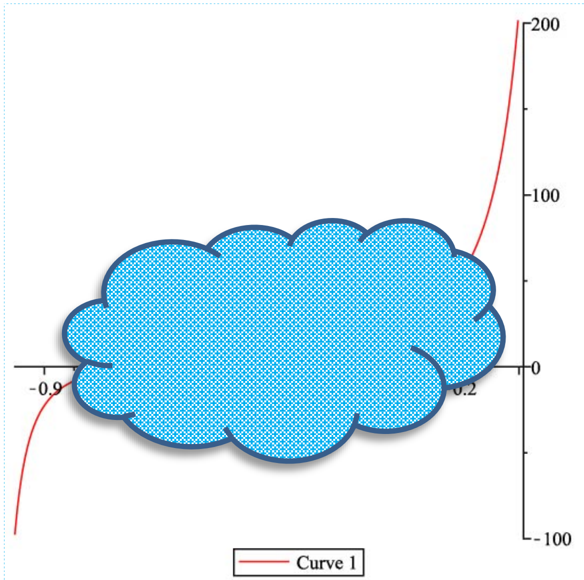
"MS Potential" : determined by the background

$$\text{Initial Singularity} : W_s \underset{\eta\to-\eta_0}{\sim} -\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2}$$

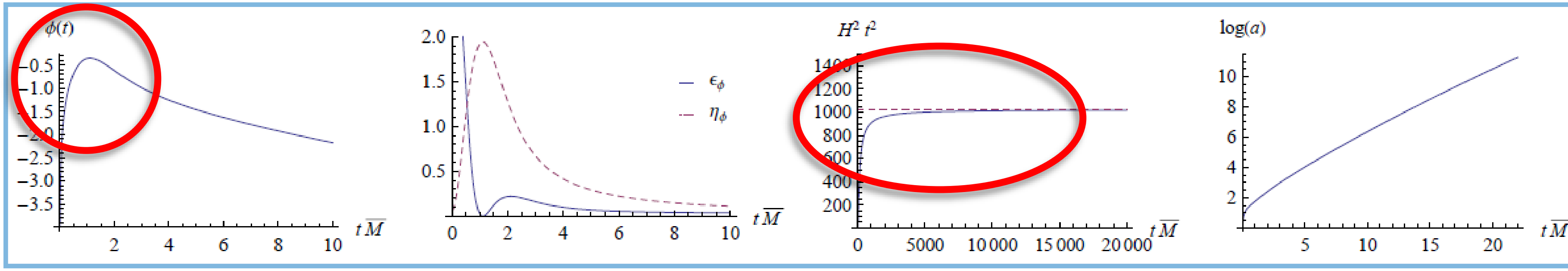
$$\text{LM Inflation} : W_s \underset{\eta\to 0}{\sim} \frac{\nu^2 - \frac{1}{4}}{\eta^2}$$

$$\left[\nu = \frac{3}{2}\,\frac{1-\gamma^2}{1-3\gamma^2}\right]$$

$$P(k) \sim k^3 \left|\frac{v(-\epsilon)}{z(-\epsilon)}\right|^2$$



Numerical Power Spectra



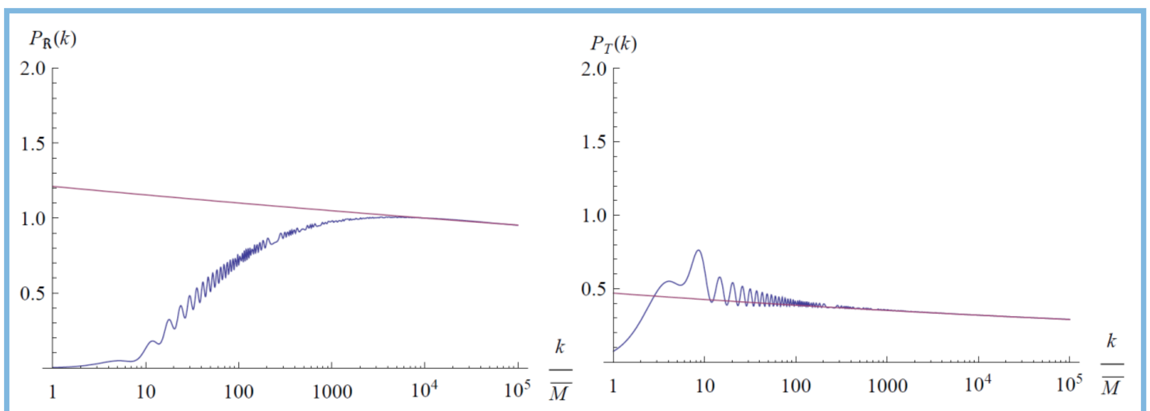
Key features:

1. Harder "kicks" make φ reach **later** the attractor
2. Even with mild kicks the **time scale** is $10^3 - 10^4$ in t M !
3. η re-equilibrates slowly

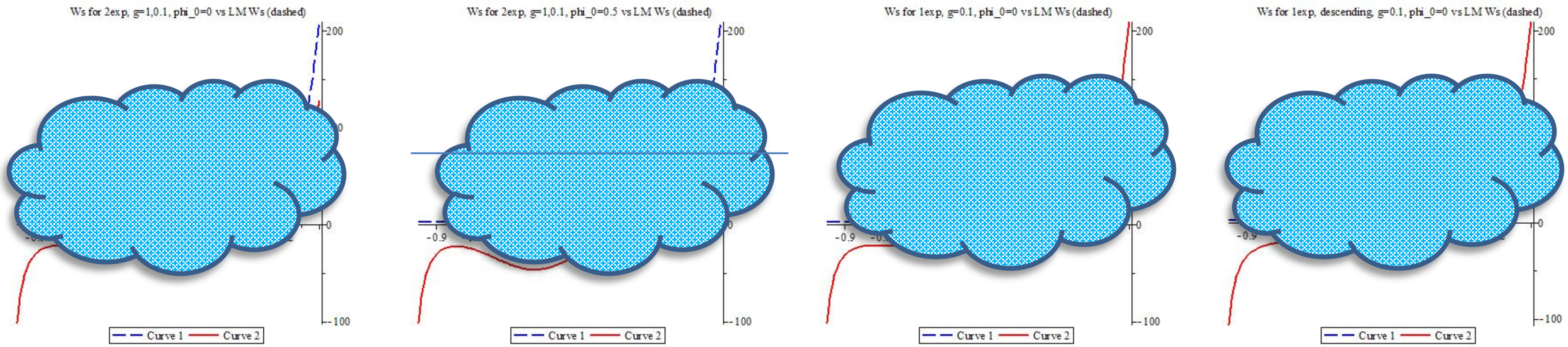
$$\epsilon_\phi \equiv - \frac{\dot{H}}{H^2} \, , \qquad \eta_\phi \equiv \frac{V_{\phi\phi}}{V}$$

$$P_{S,T} \sim \int \frac{dk}{k} \, k^{n_{S,T}-1}$$

$$n_S - 1 = 2(\eta_\phi - 3\epsilon_\phi) \, ,$$
$$n_T - 1 = -2\epsilon_\phi$$

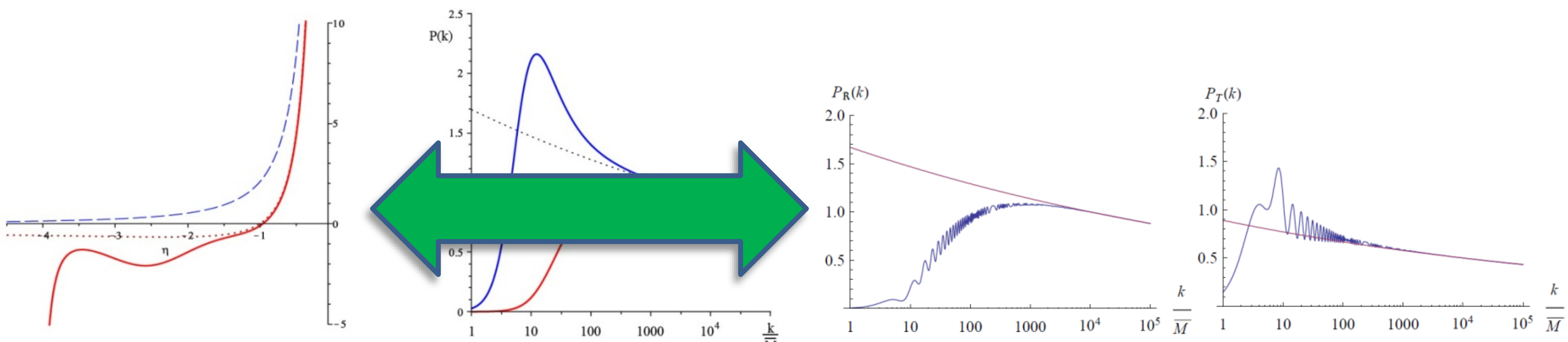


Analytic Power Spectra



WKB:

$$v_k(-\epsilon) \sim \frac{1}{\sqrt[4]{|W_s(-\epsilon) - k^2|}} \exp\left(\int_{-\eta^\star}^{-\epsilon} \sqrt{|W_s(y) - k^2|}\, dy\right)$$



WIGGLES: cfr. Q.M. resonant transmission

An Observable Window?

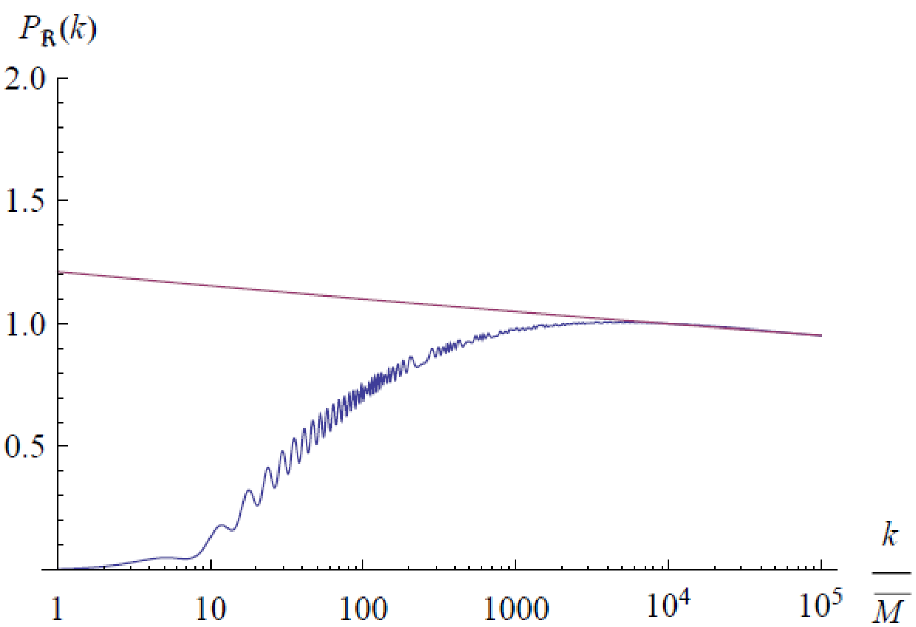
Multipole moments of the angular power spectrum (large angular scales ($\ell \leq 30$), are given to an excellent approximation by (Mukhanov)

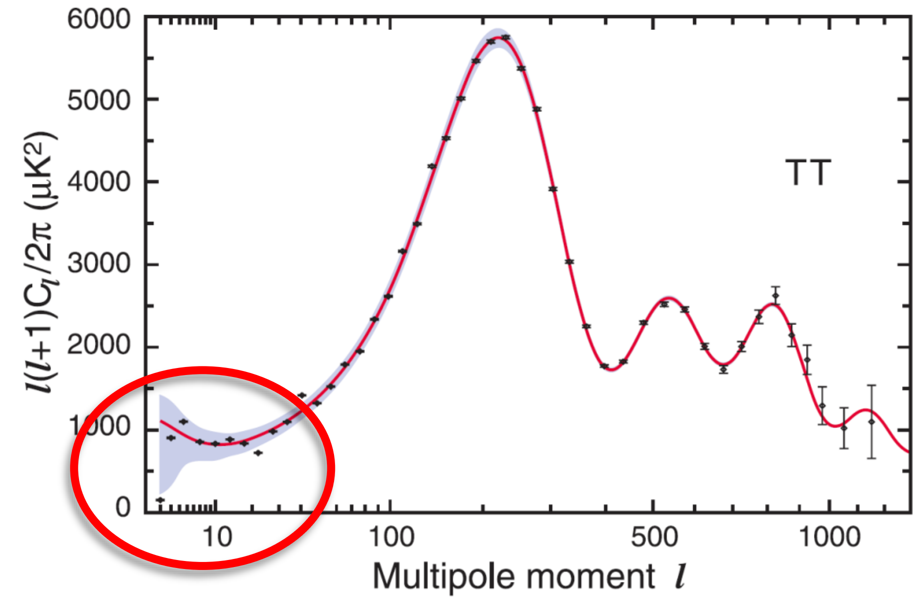
$$C_\ell = \frac{2}{9\pi} \int \frac{dk}{k} \mathcal{P}_\mathcal{R}(k) \; j_\ell^2[k(\eta_0 - \eta_r)] \; ,$$

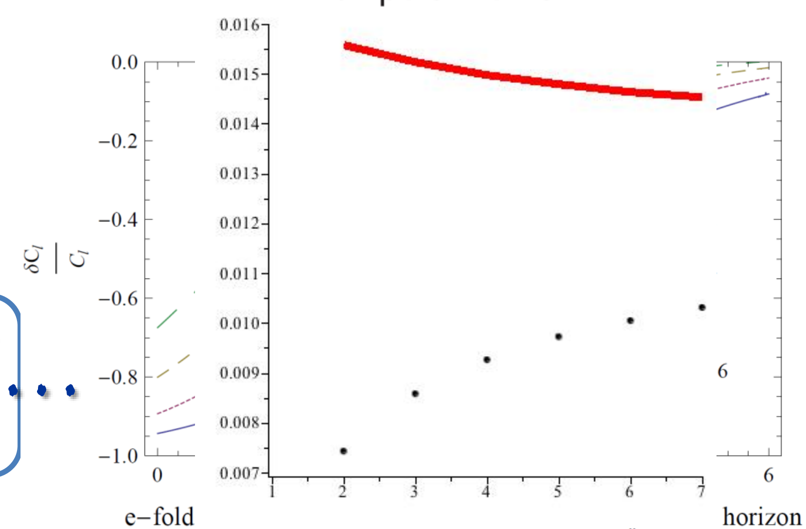
where $\eta_0 - \eta_r$ denotes our present comoving distance to last scattering surface.

- If inflation had started within 6-7 e–folds of our present horizon exit, climbing would bring about a noticeable drop in power at the largest angular scales.
- That would become more significant the closer the climbing phase were to the exit of our current horizon.

WMAP9/Planck power spectrum :





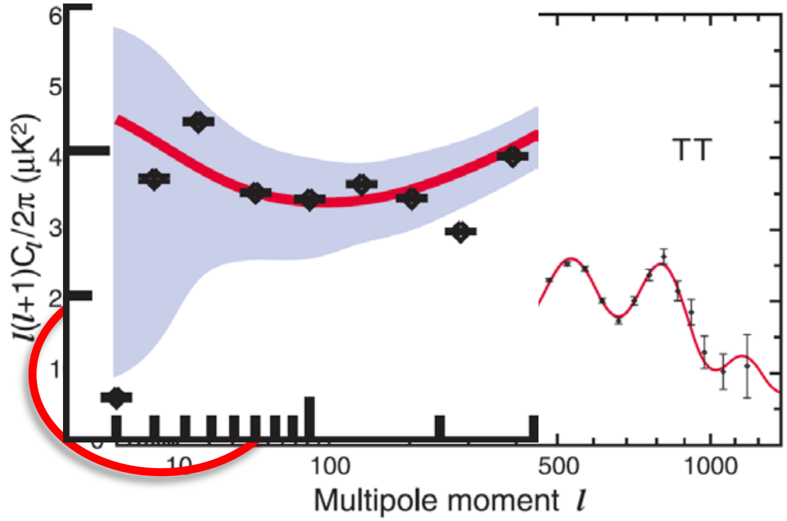


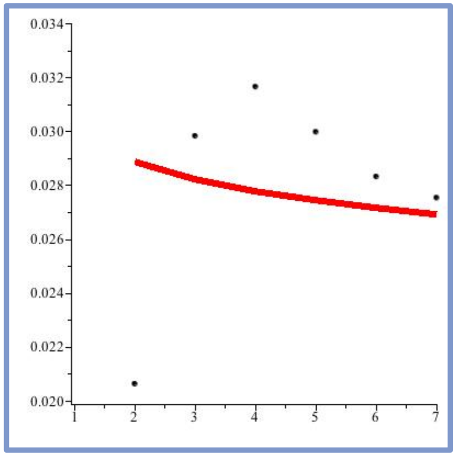
❖ NOTE: But with a harder "kick" …

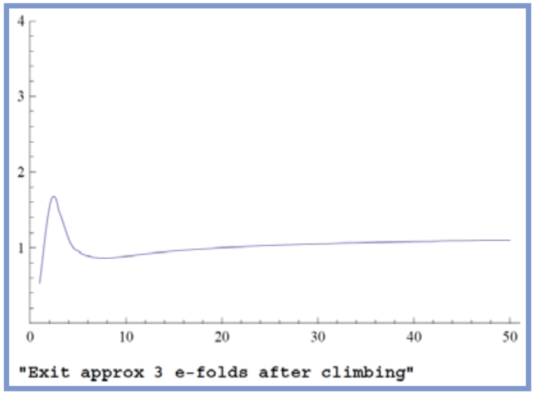
$$\left|\frac{\Delta C_\ell}{C_\ell}\right| = \sqrt{\frac{2}{2\ell+1}}$$

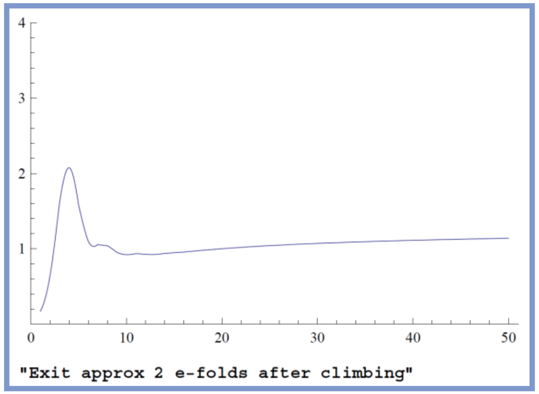
Qualitatively the low-k tail

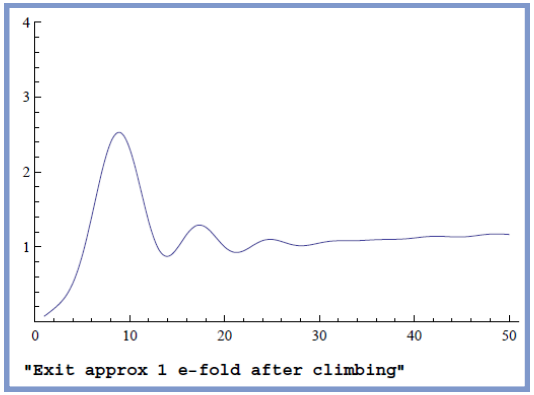
(E.D, Kitazawa, Patil, Sagnotti, in progress)











Another way of presenting the results in slide 11

2 parameters to adjust : "hardness" of kick & time of horizon exit

Kasner approach

Search for approximate Kasner-like solutions near big-bang (t=0)

$$ds^2 = -dt^2 + \sum_{i=1}^{d} t^{2a_i} dx_i^2 \ , \ \Phi = p \ln t \qquad (1)$$

The leading order e.o.m. close to big-bang reduce to

$$\sum_{i=1}^{d} a_i \ = \ 1 \ , \ \sum_{i=1}^{d} a_i^2 + \frac{1}{2} p^2 \ = \ 1$$

whereas for the exponential potential V = α exp (Δ φ)
the descending solution exists if Δ p > - 2 . Then we find:

- for asymmetric metric there is always a descending solution
- for the symmetric (FRW) case a_i = a, the descending solution exists if

$$\Delta \ > \ \sqrt{\frac{2d}{d-1}} \equiv \Delta_c$$,in agreement with the exact solution

The method can be used to analyze the climbing behaviour of any lagrangian (and any potential). Some results (FRW case):

- Higher-derivative corrections typically spoil the climbing behaviour. Specific operators preserve it. Quartic order:

$$S = \frac{1}{2} \int d^{d+1}x \sqrt{-g}\eta \left\{ R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{(d-2)(d-3)}{4d(d-1)} \left[(\nabla\phi)^4 - 2\sqrt{\frac{2(d-1)}{d}} (\nabla\phi)^2 \Box\phi \right] \right\}$$

- Most other higher-derivatives spoils it. Ex: DBI

$$S = \int d^{d+1}x \sqrt{-g} \left[R - \sqrt{1 + (\partial\phi)^2} - V(\phi) \right]$$

The scalar close to big-bang is force to slow-down

$$\phi \simeq \phi_0 \pm (t - \frac{p^2 t^5}{10}) \,, \quad \text{where}$$

$$a = \frac{2}{d} \quad , \quad |p| = \frac{d}{4(d-1)}$$

The scalar potential $V = \alpha \exp(\Delta\,\varphi)$ is now regular for both descending and climbing solution, for any Δ .

Examples with no big-bang

Consider the potentials with asymptotic behaviour

$$V = 2\tilde{\alpha}_1 e^{\gamma_1 \Phi} + 2\tilde{\alpha}_2 e^{-\gamma_2 \Phi}$$

For:

- $\gamma_1, \gamma_2 < \Delta_c$ ⇒ Kasner/FRW solutions starting on either side $\pm\infty$ of the minimum
- $\gamma_1 < \Delta_c$, $\gamma_2 > \Delta_c$ ⇒ scalar starts near big-bang necessarily on the flat side $-\infty$

Moreover, for $\gamma_1\gamma_2 \leq \frac{1}{8}\Delta_c^2$ the scalar is exponentially damped to the minimum, whereas for $\gamma_1\gamma_2 > \frac{1}{8}\Delta_c^2$ there is damping plus oscillations.

For $\gamma_1, \gamma_2 > \Delta_c$, no singular solutions anymore. Scalar forced to stay close to minimum. No big-bang !

Summary & Outlook

- **BRANE SUSY BREAKING** $(d \leq 10)$: "critical" exponential potentials
- **"HARD"** exponential of BSB **+** **"MILD"** exponential (for inflation) :

❖ WITH "short" inflation (~ 60 e-folds) :

- **WIDE IR depression** of scalar spectrum (~ 6 e-folds)
- *[MILDER IR enhancement of tensor spectrum]*
- **LARGE quadrupole** depression & **qualitatively** next few **multipoles !**
- *[LARGE CLASS of integrable potentials with climbing (Fre,Sagnotti,Sorin, to appear)]*

BISPECTRUM ?

- Kasner approach used to analyze climbing for various Models, confirms and extend previous analysis.

Multumesc pentru atentie

Extra slides

More analytical spectra

Analogy with QM allows us to anticipate :

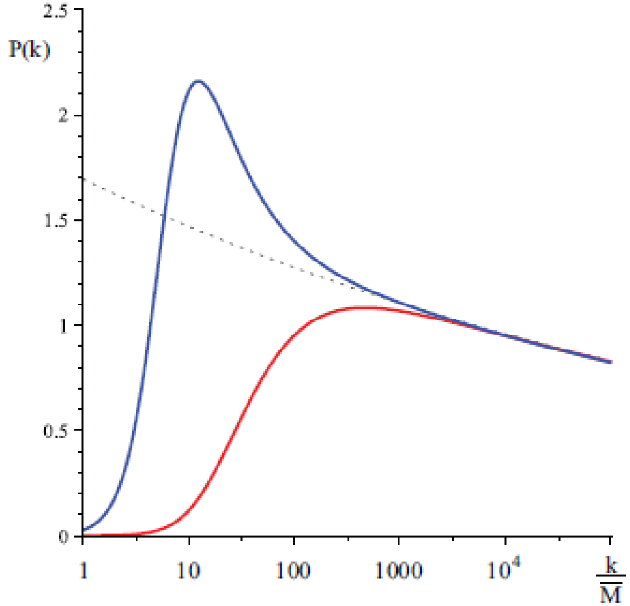
- oscillations for intermediate momenta k.
- supression of the power spectrum for small k.

There are interesting deformations of the attractor MS potential that analytically capture gross features of the actual MS scalar and tensor potentials:

$$W_S = \frac{\nu^2 - \frac{1}{4}}{\eta^2} \left[c \left(1 + \frac{\eta}{\eta_0} \right) \; + \; (1-c) \left(1 + \frac{\eta}{\eta_0} \right)^2 \right] \; ,$$

They combine the proper LM late–time behavior, a single zero and an almost flat region.

v_k = Coulomb wave functions.



Analytic scalar (red) and tensor (blue) spectra vs attractor spectrum (dotted).

$$P_R(k) \sim \frac{(k\,\eta_0)^3 \exp\left(\frac{\pi\left(\frac{c}{2}-1\right)\left(\nu^2-\frac{1}{4}\right)}{\sqrt{(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)}}\right)}{\left|\Gamma\left(\nu+\frac{1}{2}+\frac{i\left(\frac{c}{2}-1\right)\left(\nu^2-\frac{1}{4}\right)}{\sqrt{(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)}}\right)\right|^2 \left[(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)\right]^{\nu}}\,.$$

Scales

- BSB potential:

$$T_{10} = \frac{1}{(\alpha')^5} \to T_4 = \frac{1}{(\alpha')^2}\left(\frac{R}{\sqrt{\alpha'}}\right)^6 = (\overline{M})^4$$

- Attractor Power spectra:

$$\begin{aligned} P_S(k) &= \frac{1}{16\pi G_N\,\epsilon}\left(\frac{H_\star}{2\pi}\right)^2\left(\frac{k}{a\,H_\star}\right)^{n_S-1} \\ P_T(k) &= \frac{1}{\pi G_N\,\epsilon}\left(\frac{H_\star}{2\pi}\right)^2\left(\frac{k}{a\,H_\star}\right)^{n_T-1} \\ n_S &= 1-6\epsilon+2\eta \qquad n_T = 1-2\,\epsilon \\ \epsilon &= 8\pi G_N\left(\frac{V'}{V}\right)^2, \qquad \eta = 16\pi G_N\left(\frac{V''}{V}\right)^2 \end{aligned}$$

- COBE normalization & bounds on ϵ:

$$H_\star \approx 10^{15} \times (\epsilon)^{\frac{1}{2}}\ GeV$$

$$\bar{M} \approx 6.5\ 10^{16} \times (\epsilon)^{\frac{1}{4}}\ GeV$$

$$10^{-4} < \frac{P_T}{P_S} < 1.28 \to 10^{-5} < \epsilon < 0.08$$

$$3.5\ 10^{15}\ GeV < \overline{M} < 3\ 10^{16}\ GeV$$

$$3\ 10^{12}\ GeV < H_\star < 3.4\ 10^{14}\ GeV$$