# Solutii Almost-BPS in spatiul-timp Taub-NUTmulti-centru

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# Introducere

- scopul lucrarii este de a construi solutii almost-BPS in 4 dimensiuni descriind un sistem echivalent cu un numar arbitrar de inele negre extremale coliniare intr-un spatiu-timp multi-centru Taub-NUT, coliniare cu una dintre gaurile negre rotitoare aflate intr-unul dintre centri spatiu-timpului in N=2 SUGRA
- in literatura au fost extensiv studiate solutiile BPS, inclusiv in 4 dimensiuni
- solutiile almost-BPS se obtin din cele BPS, ca o generalizare a acestora, schimband semnul in ecuatia campurilor electrice si magnetice, astfel incat campurile dipolare sa fie anti-self-dual
- se constriesc solutii de microstari dotate cu regularitate si horizonless pentru a testa ipoteza 'fuzzball'

# Punerea problemei

- se porneste cu o teorie SUGRA in 11 dimensiuni (N=2) cu cate trei M-branes cu sarcini M2 (electrice) si M5 (magnetice)
- se ajunge prin compactificare la un spatiu multi-centru Taub-NUT cu supersimetrie N=2

Ansatz-ul pentru metrica si campul de etalonare pentru sarcinile M2 este:

$$ds_{11}^2 = -(Z_1 Z_2 Z_3)^{-2/3}(dt+k) + (Z_1 Z_2 Z_3)^{1/3} ds_4^2 + \left(\frac{Z_2 Z_3}{Z_1^2}\right)^{1/3}(dx_1^2 + dx_2^2) + \left(\frac{Z_1 Z_3}{Z_2^2}\right)^{1/3}(dx_3^2 + dx_4^2) + \left(\frac{Z_1 Z_2}{Z_3^2}\right)^{1/3}(dx_5^2 + dx_6^2) \quad (1)$$

$$C^{(3)} = \left(a^1 - \frac{dt+k}{Z_1}\right) \wedge dx_1 \wedge dx_2 + \left(a^2 - \frac{dt+k}{Z_2}\right) \wedge dx_3 \wedge dx_4 + \left(a^3 - \frac{dt+k}{Z_3}\right) \wedge dx_5 \wedge dx_6 \quad (2)$$

## Punerea problemei

Obtinerea solutiilor se face de fapt in contextul metricii Taub-NUT:

$$d^2s_4 = (V^m)^{-1}(d\psi + A) + V^m ds_3^2 \tag{3}$$

cu un potential Gibbons-Hawking

$$V^m = h + \frac{q}{r} + \frac{q'}{r'}, \qquad A = qcos\theta d\phi, \qquad d^2s_3 = dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \tag{4}$$

# Punerea problemei

- solutiile almost-BPS sunt date de urmatoarele ecuatii obtinute prin metode heuristice si constrangeri aplicate metricii in 11 dimensiuni

$$\Theta^{(I)} = - *_4 \Theta^{(I)} \tag{5}$$

$$d *_4 dZ_I = \frac{C_{IJK}}{2} \Theta^{(I)} \wedge \Theta^{(I)} \tag{6}$$

$$dk - *_4 dk = Z_I \Theta^{(I)} \tag{7}$$

unde

- $Z_I$ sunt warp-factorii metricii
- $\Theta^{(I)} = da^I$ sunt campurile magnetice dipolare ale teoriei (M5)
- k este momentul cinetic orbital

# Notatii

- $V^u = h + \frac{q}{r}$, Taub-NUT uni-centru
- $V^m = h + \frac{q}{r} + \frac{q'}{r'}$, Taub-NUT multi-centru
- $\Sigma_i = \sqrt{r^2 + a_i^2 - 2ra_i cos\theta}$
- $d_i$ sunt dipoli magnetici
- $K^{(I)} = \sum_i \frac{d_i^{(I)}}{\Sigma_i}$, functii harmonice caracterizand campurile dipolar magnetice
- $L_I = l_I + \sum_i \frac{Q_i^{(I)}}{\Sigma_i}$, functii harmonice caracterizand campurile electrice

# Forma solutiilor generice

$$\Theta^{(i)} = d[K^{(i)}(d\psi + A) + b^{(I)}] \quad (8)$$

$$Z_I = L_I + \frac{|\epsilon_{IJK}|}{2} \sum_{j,k} (h + \frac{qr}{a_j a_k} + \frac{q' r'}{a_j a_k}) \frac{d_j^{(J)} d_k^{(K)}}{\Sigma_j \Sigma_k} \quad (9)$$

$$k = \mu(d\psi + A) + \omega \quad (10)$$

# Ecuatiile in momentul cinetic orbital

Se porneste de la ecuatia:

$$dk - *_4 dk = Z_I \Theta^{(I)} \tag{11}$$

si se obtine:

$$d(V^m \mu) + *_3 d\omega = V^m Z_I dK^{(I)} = V^m \sum_i l_I d_i^{(I)} d\frac{1}{\Sigma_i} +$$
$$+(h+\frac{q}{r}+\frac{q'}{r'}) \sum_{i,j} Q_i^{(I)} d_j^{(I)} \frac{1}{\Sigma_i} d\frac{1}{\Sigma_j} + \frac{|\epsilon_{IJK}|}{2} \sum_{i,j,k} d_i^{(I)} d_j^{(J)} d_k^{(K)} [h^2 + \frac{hq}{r} + \frac{hqr}{a_j a_k} +$$
$$+ \frac{q^2}{a_j a_k} + \frac{hq'}{r'} + \frac{hq' r'}{a_j a_k} + \frac{qq' r}{a_j a_k r'} + \frac{qq' r'}{a_j a_k r} + \frac{q'^2}{a_j a_k}) \frac{1}{\Sigma_j \Sigma_k} d\frac{1}{\Sigma_i} \tag{12}$$

# Ecuatiile desfasurate ale momentului cinetic orbital

$$d(V^m\mu_i^{(1)})+*_3d\omega_i^{(1)}=V^m d\frac{1}{\Sigma_i} \tag{13}$$

$$d(V^m\mu_i^{(2)})+*_3d\omega_i^{(2)}=\frac{1}{\Sigma_i}d\frac{1}{\Sigma_i}(i\neq 0) \tag{14}$$

$$d(V^m\mu_{ij}^{(3)})+*_3d\omega_{ij}^{(3)}=\frac{1}{\Sigma_i}d\frac{1}{\Sigma_j}(i\neq j,,j\neq 0) \tag{15}$$

$$d(V^m\mu_i^{(4)})+*_3d\omega_i^{(4)}=\frac{1}{r\Sigma_i}d\frac{1}{\Sigma_i}(i\neq 0) \tag{16}$$

$$d(V^m\mu_i^{(4')})+*_3d\omega_i^{(4')}=\frac{1}{r'\Sigma_i}d\frac{1}{\Sigma_i}(i\neq 0) \tag{17}$$

# Ecuatiile desfasurate ale momentului cinetic orbital

$$d(V^m \mu_{ij}^{(5)}) + *_3 d\omega_{ij}^{(5)} = \frac{1}{r\Sigma_i} d\frac{1}{\Sigma_j}(i \neq j, j \neq 0) \tag{18}$$

$$d(V^m \mu_{ij}^{(5')}) + *_3 d\omega_{ij}^{(5')} = \frac{1}{r'\Sigma_i} d\frac{1}{\Sigma_j}(i \neq j, j \neq 0) \tag{19}$$

$$d(V^m \mu_{ijk}^{(6)}) + *_3 d\omega_{ijk}^{(6)} = T_{ijk}^{(1)}(i, j, k \neq 0) \tag{20}$$

$$d(V^m \mu_{ijk}^{(7)}) + *_3 d\omega_{ijk}^{(7)} = T_{ijk}^{(2)}(i, j, k \neq 0) \tag{21}$$

$$d(V^m \mu_{ijk}^{(8)}) + *_3 d\omega_{ijk}^{(8)} = T_{ijk}^{(3)}(i, j, k \neq 0) \tag{22}$$

$$d(V^m \mu_{ijk}^{(9)}) + *_3 d\omega_{ijk}^{(9)} = T_{ijk}^{(4)}(i, j, k \neq 0) \tag{23}$$

# Ecuatiile desfasurate ale momentului cinetic orbital

$$d(V^m \mu_{ijk}^{(10)}) + *_3 d\omega_{ijk}^{(10)} = T_{ijk}^{(5)}(i,j,k \neq 0) \tag{24}$$

$$d(V^m \mu_{ijk}^{(11)}) + *_3 d\omega_{ijk}^{(11)} = T_{ijk}^{(6)}(i,j,k \neq 0) \tag{25}$$

with

$$T_{ijk}^{(1)} = \frac{1}{\Sigma_i}\frac{1}{\Sigma_j} d\frac{1}{\Sigma_k} + \frac{1}{\Sigma_k}\frac{1}{\Sigma_i} d\frac{1}{\Sigma_j} + \frac{1}{\Sigma_j}\frac{1}{\Sigma_k} d\frac{1}{\Sigma_i} \tag{26}$$

$$T_{ijk}^{(2)} = \frac{1}{a_i a_j \Sigma_i}\frac{1}{\Sigma_j} d\frac{1}{\Sigma_k} + \frac{1}{a_k a_i \Sigma_k}\frac{1}{\Sigma_i} d\frac{1}{\Sigma_j} + \frac{1}{a_j a_k \Sigma_j}\frac{1}{\Sigma_k} d\frac{1}{\Sigma_i} \tag{27}$$

# Ecuatiile desfasurate ale momentului cinetic orbital

$$T^{(3)}_{ijk}=(\frac{1}{r}+\frac{r}{a_i a_j})\frac{1}{\Sigma_i}\frac{1}{\Sigma_j}d\frac{1}{\Sigma_k}+(\frac{1}{r}+\frac{r}{a_i a_j})\frac{1}{\Sigma_k}\frac{1}{\Sigma_i}d\frac{1}{\Sigma_j}+(\frac{1}{r}+\frac{r}{a_i a_j})\frac{1}{\Sigma_j}\frac{1}{\Sigma_k}d\frac{1}{\Sigma_i} \tag{28}$$

$$T^{(4)}_{ijk}=(\frac{1}{r'}+\frac{r'}{a_i a_j})\frac{1}{\Sigma_i}\frac{1}{\Sigma_j}d\frac{1}{\Sigma_k}+(\frac{1}{r'}+\frac{r'}{a_i a_j})\frac{1}{\Sigma_k}\frac{1}{\Sigma_i}d\frac{1}{\Sigma_j}+(\frac{1}{r'}+\frac{r'}{a_i a_j})\frac{1}{\Sigma_j}\frac{1}{\Sigma_k}d\frac{1}{\Sigma_i} \tag{29}$$

$$T^{(5)}_{ijk}=\frac{r}{r'}T^{(2)}_{ijk} \tag{30}$$

$$T^{(6)}_{ijk}=\frac{r'}{r}T^{(2)}_{ijk} \tag{31}$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$V^u \mu_i^{(1)}(r,\theta,\phi) = \frac{V^u}{2\Sigma_i}, \quad \omega_i^{(1)}(r,\theta,\phi) = \frac{h}{2}\frac{rcos\theta - a_i}{\Sigma_i}d\phi + \frac{q}{2}\frac{r - a_i cos\theta}{a_i\Sigma_i}d\phi \tag{32}$$

$$V^u \mu_i^{(2)}(r,\theta,\phi) = \frac{1}{2\Sigma_i^2}, \qquad \omega_i^{(2)}(r,\theta,\phi) = 0 \tag{33}$$

$$V^u \mu_{ij}^{(3)}(r,\theta,\phi) = \frac{1}{2}\frac{1}{\Sigma_i\Sigma_j}, \quad \omega_{ij}^{(3)}(r,\theta,\phi) = \frac{r^2 + a_i a_j - (a_i + a_j) rcos\theta}{2(a_j - a_i)\Sigma_i\Sigma_j}d\phi \tag{34}$$

$$V^u \mu_i^{(4)}(r,\theta,\phi) = \frac{cos\theta}{2a_i\Sigma_i^2}, \qquad \omega_i^{(4)}(r,\theta,\phi) = \frac{rsin^2\theta}{2a_i\Sigma_i^2}d\phi \tag{35}$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$V^u\mu_{ij}^{(5)}(r,\theta,\phi) = \frac{r^2 + a_i a_j - 2a_j r cos\theta}{2a_j(a_i - a_j) r \Sigma_i \Sigma_j},$$
$$\omega_{ij}^{(5)}(r,\theta,\phi) = \frac{r(a_i + a_j cos^2\theta) - (r^2 + a_i a_j) cos\theta}{2a_j(a_i - a_j)\Sigma_i \Sigma_j} d\phi \quad (36)$$

$$V^u\mu_{ijk}^{(6)}(r,\theta,\phi) = \frac{1}{\Sigma_i\Sigma_j\Sigma_k} \qquad \omega_{ijk}^{(6)}(r,\theta,\phi) = 0 \quad (37)$$

$$V^u\mu_{ijk}^{(7)}(r,\theta,\phi) = \frac{r cos\theta}{a_i a_j a_k \Sigma_i\Sigma_j\Sigma_k} \qquad \omega_{ijk}^{(7)}(r,\theta,\phi) = \frac{r^2 sin^2\theta}{a_i a_j a_k \Sigma_i\Sigma_j\Sigma_k} d\phi \quad (38)$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$V^u \mu^{(8)}_{ijk}(r,\theta,\phi) = \frac{r^2(a_i + a_j + a_k) + a_i a_j a_k}{2 a_i a_j a_k r \Sigma_i \Sigma_j \Sigma_k} \tag{39}$$

$$\omega^{(8)}_{ijk}(r,\theta,\phi) = \frac{r^3 + r(a_i a_j + a_i a_k + a_j a_k) - (r^2(a_i + a_j + a_k) + a_i a_j a_k) cos\theta}{2 a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k} \tag{40}$$

$$V^m \mu^{(4')}_i(r,r',\theta,\phi) = \frac{r' cos\theta - a_i sin\theta}{2 a_i^2 \Sigma_i^2}, \quad \omega^{(4')}_i(r,r',\theta,\phi) = \frac{r - r' cos\theta}{2 a_i \Sigma_i^2} d\phi \tag{41}$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$V^m \mu_{ij}^{(5')}(r, r', \theta, \phi) = \frac{r'}{2a_i a_j \Sigma_i \Sigma_j},$$

$$\omega_{ij}^{(5')}(r, r', \theta, \phi) = \frac{r^2 + r'^2 + a_i a_j - (a_i + a_j)(r - r')cos\theta}{2a_i a_j \Sigma_i \Sigma_j} d\phi \quad (42)$$

$$V^m \mu_{ijk}^{(9)}(r, r', \theta, \phi) = \frac{(r' + a_i sin\theta)(r - a_j cos\theta)(r - a_k cos\theta)}{a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k} + \frac{(r' + a_j sin\theta)(r - a_k cos\theta)(r - a_i cos\theta)}{a_i a_j a_k \Sigma_i \Sigma_j^3 \Sigma_k} + \frac{(r' + a_k sin\theta)(r - a_i cos\theta)(r - a_j cos\theta)}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k^3} \quad (43)$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$\omega^{(9)}_{ijk}(r,r',\theta,\phi) = (\frac{r'^4 + r^4 + a_j a_k r r' sin\theta}{a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k} + \\ + \frac{r'^4 + r^4 + a_i a_k r r' sin\theta}{a_i a_j a_k \Sigma_i \Sigma_j^3 \Sigma_k} + \\ + \frac{r'^4 + r^4 + a_i a_j r r' sin\theta}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k^3}) d\phi \quad (44)$$

$$V^m \mu^{(10)}_{ijk}(r,r',\theta,\phi) = \frac{r' cos\theta}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k} + \frac{r'^2 cos\theta (r - 2a_i cos\theta)}{a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k} + \\ + \frac{r'^2 cos\theta (r - 2a_j cos\theta)}{a_i a_j a_k \Sigma_i \Sigma_j^3 \Sigma_k} + \frac{r'^2 cos\theta (r - 2a_k cos\theta)}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k^3} \quad (45)$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$\omega^{(10)}_{ijk}(r,r',\theta,\phi) = (\frac{2r'^2 sin^2\theta}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k} + \frac{r'^2 sin^2\theta(r^2 - 2a_i^2 cos\theta)}{a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k} + \\ + \frac{r'^2 sin^2\theta(r^2 - 2a_j^2 cos\theta)}{a_i a_j a_k \Sigma_i \Sigma_j^3 \Sigma_k} + \\ + \frac{r'^2 sin^2\theta(r^2 - 2a_k^2 cos\theta)}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k^3}) d\phi \quad (46)$$

$$V^m \mu^{(11)}_{ijk}(r,r',\theta,\phi) = \frac{rr'^2 - a_i a_j a_k cos\theta}{a_i a_j a_k r^2 \Sigma_i \Sigma_j \Sigma_k} + \frac{r'^3 sin\theta + a_i a_j a_k}{2a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k} + \\ + \frac{r'^3 sin\theta + a_i a_j a_k}{2a_i a_j a_k \Sigma_i \Sigma_j^3 \Sigma_k} + \frac{r'^3 sin\theta + a_i a_j a_k}{2a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k^3} \quad (47)$$

# Solutiile ecuatiilor in $\mu$ si $\omega$

$$\omega^{(11)}_{ijk}(r,r',\theta,\phi) = \Big(\frac{r'^2 cos\theta}{a_i a_j a_k \Sigma_i \Sigma_j \Sigma_k} + \frac{r^4 + r'^4 cos\theta + r'^2 a_j a_k}{2a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k} + \\ + \frac{r^4 + r'^4 cos\theta + r'^2 a_i a_k}{2a_i a_j a_k \Sigma_i \Sigma_j^3 \Sigma_k} + \frac{r^4 + r'^4 cos\theta + r'^2 a_i a_j}{2a_i a_j a_k \Sigma_i^3 \Sigma_j \Sigma_k}\Big) d\phi \quad (48)$$

$$\mu^{shift}(r,r',\theta,\phi) = \frac{q' r cos\theta}{r'^2}, \qquad \omega^{shift}(r,r',\theta,\phi) = \frac{r'^2 sin\theta}{r} d\phi \quad (49)$$

# Solutiile ecuatiilor omogene

$$\mu^{(12)}(r, r', \theta, \phi) = \frac{1}{V^m}(m_0 + \sum_i \frac{m_i}{\Sigma_i} + \sum_i \alpha_i \frac{cos\theta_i}{\Sigma_i^2} + \frac{\beta}{r'}) \qquad (50)$$

$$\omega^{(12)}(r, r', \theta, \phi) = \kappa d\phi - \sum_i m_i cos\theta_i d\phi + \sum_i \alpha_i \frac{r^2 sin^2\theta}{\Sigma_i^3} d\phi - \frac{\beta}{r'^2} d\phi \quad (51)$$

# Solutia completa in $\mu$

$$
\begin{aligned}
\mu(r,r',\theta,\phi) = &\sum_i (l_I d_i^{(I)} \mu_i^{(1)} + Q_i^{(I)} d_i^{(I)} (h\mu_i^{(2)} + q\mu_i^{(4)})) + \\
&+ \sum_{i\neq j} Q_i^{(I)} d_j^{(I)} (h\mu_{ij}^{(3)} + q\mu_{ij}^{(5)}) + \sum_i Q_i^{(I)} d_i^{(I)} q' \mu_i^{(4')} + \sum_{i\neq j} Q_i^{(I)} d_j^{(I)} q' \mu_{ij}^{(5')} + \\
&+ \sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} (h^2 \mu_{ijk}^{(6)} + (q^2 + q'^2)\mu_{ijk}^{(7)} + hq\mu_{ijk}^{(8)}) + \\
&+ \sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} (q' h \mu_{ijk}^{(9)} + qq'(\mu_{ijk}^{(10)} + \mu_{ijk}^{(11)})) + \mu_{shift} + \mu^{(12)}
\end{aligned} \tag{52}
$$

# Solutia completa in $\omega$

$$\omega(r,r',\theta,\phi)=\sum_i(l_I d_i^{(I)}\omega_i^{(1)}+Q_i^{(I)}d_i^{(I)}q\omega_i^{(4)})+\sum_{i\neq j}Q_i^{(I)}d_j^{(I)}(h\omega_{ij}^{(3)}+q\omega_{ij}^{(5)})+$$
$$+\sum_i Q_i^{(I)}d_i^{(I)}q'\omega_i^{(4')}+\sum_{i\neq j}Q_i^{(I)}d_j^{(I)}q'\omega_{ij}^{(5')}+$$
$$+\sum_{i,j,k}d_i^{(1)}d_j^{(2)}d_k^{(3)}(h^2\omega_{ijk}^{(6)}+(q^2+q'^2)\omega_{ijk}^{(7)}+hq\omega_{ijk}^{(8)})+$$
$$+\sum_{i,j,k}d_i^{(1)}d_j^{(2)}d_k^{(3)}(q'h\omega_{ijk}^{(9)}+qq'(\omega_{ijk}^{(10)}+\omega_{ijk}^{(11)}))+\omega_{shift}+\omega^{(12)} \quad (53)$$

# Regularitatea solutiilor

- $\omega$ poate avea singularitati de tip stringuri Dirac-Misner si acest lucru conduce la CTC
- pentru a elimina aceste singularitati impunem conditia $\omega = 0$ pe axa z
- de asemenea in apropierea polilor functiilor harmonice factorii warp si momentul cinetic orbital trebuie protejati de singularitati
- aceste constrangeri fixeaza solutia ecuatiilor omogene

# Moduli fixati din solutia omogena

$$\kappa = -q\sum_i \frac{l_I d_i^{(I)}}{2a_i} - h\sum_{i\neq j} \frac{Q_i^{(I)} d_j^{(I)}}{2(a_j - a_i)} +$$
$$+\sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} \left(\frac{hq}{2a_i a_j a_k} + \frac{q' h}{a_i a_j a_k} + qq'\left(\frac{1}{a_i^2 a_j a_k} + \frac{1}{a_i a_j^2 a_k} + \frac{1}{a_i a_j a_k^2}\right)\right) \tag{54}$$

$$m_0 = -q\sum_i \frac{l_I d_i^{(I)}}{2a_i} - h\sum_i \frac{Q_0^{(I)} d_j^{(I)}}{2a_i} + q\sum_{i\neq j, i\neq 0} \frac{Q_i^{(I)} d_j^{(I)}}{2a_j(a_j - a_i)} +$$
$$+\sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} \left(\frac{hq}{2a_i a_j a_k} + qq'\left(\frac{1}{2(a_i a_j)^2} + \frac{1}{2(a_j a_k)^2} + \frac{1}{2(a_k a_i)^2}\right)\right) \tag{55}$$

# Moduli fixati din solutia omogena

$$m_i = \frac{l_I d_i^{(I)}}{2}(h + \frac{q}{a_i}) + Q_i^{(I)} d_i^{(I)} q' \frac{1}{2a_i^2} + \sum_j \frac{1}{2|a_i - a_j|}[Q_j^{(I)} d_i^{(I)}(h + \frac{q}{a_i}) -$$
$$- Q_i^{(I)} d_j^{(I)}(h + \frac{q}{a_j}) - Q_i^{(I)} d_j^{(I)} q' \frac{(a_i + a_j)^2}{2a_i^2 a_j^2}] + \frac{hq}{2}[\frac{d_i^{(1)} d_i^{(2)} d_i^{(3)}}{a_i^3} +$$
$$+ \frac{|\epsilon_{IJK}|}{2} \frac{d_i^{(I)}}{a_i} \sum_{j,k} sign(a_j - a_i) sign(a_k - a_i) \frac{d_j^{(J)} d_k^{(K)}}{a_j a_k}] + hq'[\frac{d_i^{(1)} d_i^{(2)} d_i^{(3)}}{a_i^3}] +$$
$$+ qq' \frac{|\epsilon_{IJK}|}{2} \frac{d_i^{(I)}}{a_i^2} \sum_{j,k} sign(a_j - a_i) sign(a_k - a_i) \frac{d_j^{(J)} d_k^{(K)}}{a_j a_k} \quad (56)$$

# Moduli fixati din solutia omogena

$$\beta = q\sum_i \frac{l_I d_i^{(I)}}{2a_i} + h\sum_{i\neq j}\frac{Q_i^{(I)} d_j^{(I)}}{2(a_j - a_i)} + \\ + \frac{|\epsilon_{IJK}|}{2}\sum_{i,j,k} d_i^{(I)} d_j^{(J)} d_k^{(K)} \left(\frac{qq'}{a_i^2 a_j a_k} + \frac{(hq + hq')}{a_i a_j a_k}\right) \quad (57)$$

# Regularitatea solutiilor in centri Taub-NUT

$$\mu|_{r=0,r'=0} = \sum_i (l_I d_i^{(I)} \frac{1}{2a_i} + Q_i^{(I)} d_i^{(I)} (\frac{1}{2a_i^2} + \frac{q cos\theta}{2ha_i^3} - \frac{q' sin\theta}{2ha_i^3}))+$$

$$+ \sum_{i\neq j} Q_i^{(I)} d_j^{(I)} (\frac{1}{2a_i a_j} + \frac{q}{h}\frac{1}{2a_j^2(a_i - a_j)}) + \sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} (\frac{h}{a_i a_j a_k} +$$

$$+ \frac{q' sin\theta cos^2\theta}{a_i a_j a_k} (\frac{1}{a_i^2} + \frac{1}{a_j^2} + \frac{1}{a_k^2})) + \frac{m_0 + \beta}{h} = 0 \quad (58)$$

$$\omega|_{r=0,r'=0} = (\sum_i - l_I d_i^{(I)} (\frac{h}{2} - \frac{q cos\theta}{2a_i}) + \sum_{i\neq j} Q_i^{(I)} d_j^{(I)} (\frac{h}{2(a_j - a_i)} -$$

$$- \frac{q cos\theta}{2a_j(a_i - a_j)} + \frac{q'}{2a_i a_j}) - \sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} \frac{hq cos\theta}{2a_i a_j a_k} + \kappa - m_0 cos\theta - \sum_{i\neq 0} m_i - \beta) d\phi =$$

# Regularitatea solutiilor la orizonturi

$$I_4 = Z_1 Z_2 Z_3 V^m - \mu^2 V^{m2} \tag{60}$$

si elementul de volum in jurul polului $\Sigma_i = 0$ este:

$$\sqrt{g_{H,i}} = \Sigma_i (I_4 \Sigma_i^2 sin^2\theta_i - \omega_\phi^2)^{1/2} \tag{61}$$

Dezvoltand in jurul lui $\Sigma_i = 0$ se obtine:

$$I_4 \approx -2\alpha_i d_i^{(1)} d_i^{(2)} d_i^{(3)} (h + \frac{q + q'}{a_i})^2 \frac{cos\theta_i}{\Sigma_i^5} + O(\Sigma_i^{-4}) \tag{62}$$

$$\omega_\phi \approx \Sigma_i^{-1} \tag{63}$$

# Regularitatea solutiilor la orizonturi

Conditia de regularitate a metricii si absenta CTC in afara orizontului este:

$$\alpha_i = 0 \qquad (i \geq 1) \tag{64}$$

cu acesata conditie aria orizontului in jurul lui $\Sigma_i = 0$ este:

$$A_H = 16\pi^2 q q' J_4^{1/2} \tag{65}$$

unde $J_4$ este invariantul quartic $E_{7(7)}$ dat de:

$$J_4 = \frac{1}{2}\sum_{I<J} \hat{d}_i^{(I)} \hat{d}_j^{(J)} Q_i^{(I)} Q_i^{(J)} - \frac{1}{4}\sum_I (\hat{d}_i^{(I)})^2 (Q_i^{(I)})^2 - 2\hat{d}_i^{(1)} \hat{d}_i^{(2)} \hat{d}_i^{(3)} \hat{m}_i \tag{66}$$

# Concluzii

- in aceasta lucrare au fost obtinute solutii de microstari almost-BPS pentru gauri negre coliniare cu unul dintre centri Taub-NUT intr-un Taub-NUT cu 2 centri in 4 dimensiuni
- aceste solutii permit calcularea entropiei gaurilor negre almost-BPS si permit testarea ipotezei 'fuzzball'
- conditiile de regularitate asupra solutiilor determina moduli ai solutiilor ecuatiilor omogene si fixeaza complet solutiile
- pornind de la solutii BPS au fost construite generalizari non-BPS in literatura pentru gauri negre cu 3 sarcini
- in literatura s-a facut o clasificare a solutiilor non-BPS pe baza orbitelor nilpotente ale grupurilor de simetrie in SUGRA N=8