RAPORT DE ACTIVITATE AL FAZEI

Contract nr.: PN 09370102/2009

Titlul proiectului: Elaborarea de modele teoretice și metode matematice riguroase pentru investigarea structurii materiei

Director de proiect: Dr. Aurelian Isar

Faza nr. 3/2012: Factori de formă hadronici la energii joase

Obiectivul fazei: Testarea modelului standard al particulelor la energii joase și detectarea unor posibile semnale ale fizicii dincolo de modelul standard.

Rezultate preconizate: Se vor dezvolta metode teoretice specifice regimului neperturbativ al cromodinamicii cuantice pentru descrierea factorilor de formă electromagnetici și slabi ai mezonilor pseudoscalari și calculul unor marimi de interes pentru testarea modelului standard.

Termen: 15 martie 2013

Parametrization-free determination of the shape parameters of the pion electromagnetic form factor

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in colaboration with

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Outline

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2. The pion electromagnetic form factor
3. Phenomenological and theoretical input
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5. Results
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   - higher shape parameters
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Motivation

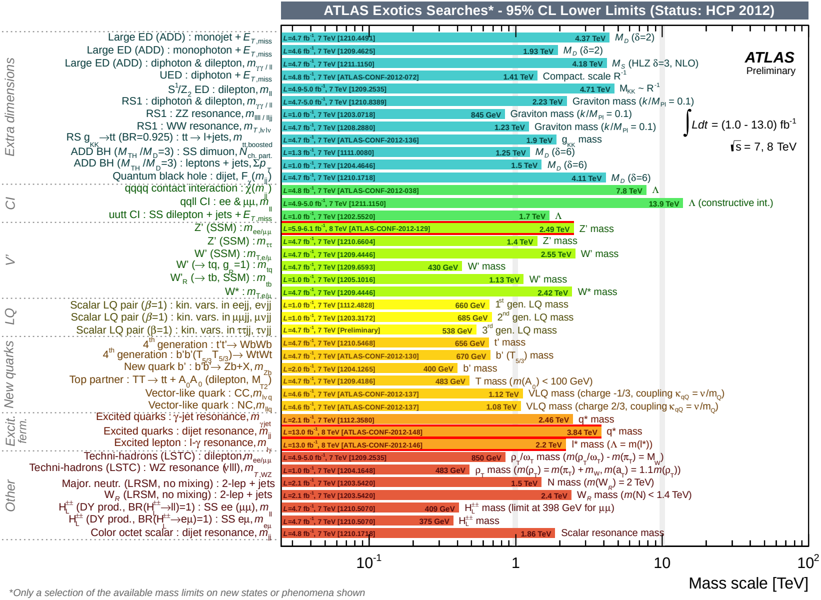
- Precision tests of Standard Model (SM) and searches for Beyond Standard Model (BSM) signals
  - direct evidence of BSM: new particles discovered in high energy collisions
  - indirect evidence of BSM: influence (through quantum fluctuations) on observables measured with high precision
    - global electroweak precision tests
    - low energy tests: muon's anomalous magnetic moment (g-2)

Search for new particles

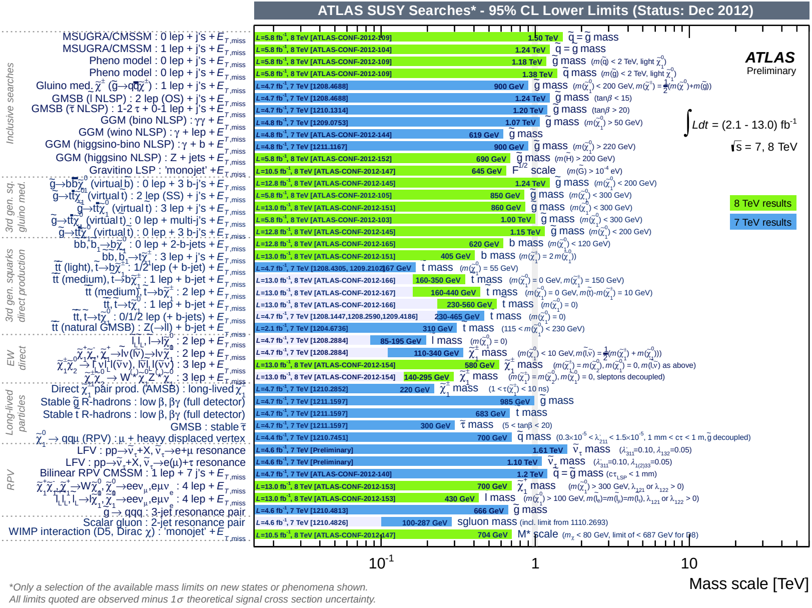
- Searches based on exotic final states signatures (two same-sign leptons, jets with high transverse momenta, large transverse missing energy, prompt energetic photons, etc)
- No particles beyond SM detected up to now
- Only lower limits on masses and upper limits on cross sections (at Tevatron, DESY, LHC)

  - a new family of quarks: $m_{t'}, m_{b'} > 670$ GeV (ATLAS)
  - a new family of heavy leptons
  - new gauge bosons Z' and W': $m_{Z'}, m_{W'} > 2.4$ TeV (ATLAS)
  - more Higgs bosons, charged or neutral
  - other exotic particles: lepto-quarks, composite or excited leptons or quarks, particles from extradimensions...
  - supersymmetric partners of SM particles: sfermions (stop), gluino, chargino, etc.

ATLAS searches for exotic particles (other than supersymmetry)



ATLAS searches for supersymmetry



- For a point-like particle Dirac equation predicts the magnetic moment:

$$\mu_{Dirac} = \frac{eh}{4\pi M}$$

- The quantum fluctuations induce an anomaly: $a_\mu = \frac{\mu_\mu}{\mu_{Dirac}} - 1 = (g_\mu - 2)/2$

- The muon magnetic moment anomaly is one of the most precisely measured observables in particle physics:

$$a_\mu^{exp} = (11659208.9 \pm 5.4_{stat} \pm 3.3_{syst}) \times 10^{-10} \qquad (PDG\ 2012)$$

- Theoretical calculation:

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{hadr} + a_\mu^{weak}$$

- The most recent theoretical predictions claim the accuracy $\delta a_\mu^{th} \sim 4.9 \times 10^{-10}$

- The Brookhaven muon g - 2 experiment finished in 2004 in different runs revealed a persisting discrepancy between theory and experiment at the 3 to 4 σ level

- Next-generation muon g-2 experiment planned at Fermilab, with a goal to reach a precision of $\delta a_\mu^{exp} \sim 1.6 \times 10^{-10}$
- Demands improved SM evaluations to fully interpret the measurement
  - The biggest theoretical uncertainties are due to the non-perturbative hadronic contributions: hadronic vacuum polarization (VP) and light-by-light scattering
  - The most important contribution to the VP is the two-pion contribution:

$$a_\mu^{\pi\pi} = \frac{\alpha^2 M_\mu^2}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{(t-4M_\pi^2)^{3/2}}{t^{7/2}} K(t) |F(t)|^2 dt$$

$$K(t) = \int_0^1 du \, \frac{(1-u)u^2}{1-u+M_\mu^2 u^2/t}$$

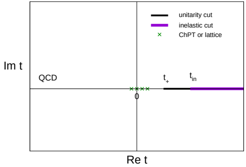
- $\langle \pi^+(p')|J_\mu^{\rm elm}|\pi^+(p)\rangle = (p+p')_\mu F(t), \quad t=(p'-p)^2$
- Describes the internal structure of the pion probed by a photon
- Taylor expansion at $t=0$: $\quad F(t) = 1+\frac{1}{6}\langle r_\pi^2\rangle t + c\, t^2 + d\, t^3 \cdots$

  $\langle r_\pi^2\rangle$: charge radius squared

  c, d: higher order shape parameters

- A unitary theoretical description not available:
  - at high $t=-Q^2<0$ (on the spacelike axis): perturbative QCD

  - at low energies: nonperturbative QCD (Chiral Perturbation Theory, lattice QCD)
  - intermediate region: interplay of perturbative and nonperturbative QCD, big uncertainties



- Causality: $F(t)$ real analytic $(F(t^*) = F^*(t))$ in the cut complex t-plane

  $t_+ = 4M_\pi^2$: the lowest unitarity threshold

- Unitarity: $\mathrm{Im}F(t+i\epsilon) = \theta(t-t_+)\,\sigma(t)f_1^*(t)F(t) + \theta(t-t_{in})\,\Sigma_{in}(t)$

  $\sigma(t) = \sqrt{1 - t_+/t}$: two particle phase space

  $f_1(t) = \frac{e^{2i\delta_1^1(t)}-1}{2i\sigma(t)}$: P-wave of $\pi\pi$ elastic scattering

  $t_{in} = (M_\pi + M_\omega)^2$: the first significant inelastic threshold

$\Rightarrow$ Fermi-Watson theorem: for $t_+ \leq t \leq t_{in}$, $\arg[F(t+i\epsilon)] = \delta_1^1(t)$,

Precise information on the pion form factor

- $\delta(t) = \delta_1^1(t)$ for $t \leq t_{in} = (M_\pi + M_\omega)^2 = (0.917 \text{ GeV})^2$ precisely known from ChPT and dispersive (Roy) equations for $\pi\pi$ amplitude Ananthanarayan et al (2001), Garcia-Martin et al (2011), Caprini, Colangelo, Leutwyler (2012)

- Recent high statistics measurements of the modulus from $e^+e^- \to \pi^+\pi^-$ or hadronic τ decays (BaBar up to 3 GeV) BaBar, KLOE, CMD-2, Belle

  $\Rightarrow$ good estimate of the integral: $\frac{1}{\pi}\int_{t_{in}}^{\infty} \rho(t)|F(t)|^2 dt = I$ for suitable weights $\rho(t)$

$\rho(t)$	I
$1/\sqrt{t}$	0.687 ± 0.028
$1/t$	0.578 ± 0.022

- Precise indirect measurements from $ep \to en\pi^+$ at several spacelike points Huber et al (2008)

t	Value [GeV^2]	$F(t)$
t_1	-1.60	$0.243 \pm 0.012^{+0.019}_{-0.008}$
t_2	-2.45	$0.167 \pm 0.010^{+0.013}_{-0.007}$

- The origin is not directly accessible. An analytic continuation is necessary.

Analytic continuation

- Dispersive representations
  - Standard dispersion relation (Cauchy integral)

    $F(t) = \frac{1}{\pi}\int_{t_+}^{\infty} \frac{\mathrm{Im}F(t'+i\epsilon)dt'}{t'-t}$ (modulo subtractions)
  - Omnès (phase) representations $\arg F(t+i\epsilon)(t) = \delta(t)$ (

    $F(t) = P(t)\exp\left(\frac{t}{\pi}\int_{t_+}^{\infty} dt' \frac{\delta(t')}{t'(t'-t)}\right)$

    $P(t)$: polynomial (accounts for zeros: $P(t_i)=0$)
  - Representation in terms of modulus

    $F(t) = B(t)\exp\left(\frac{\sqrt{t_+-t}}{\pi}\int_{t_+}^{\infty} \frac{\ln|F(t')|\,\mathrm{d}t'}{\sqrt{t'-t_+}(t'-t}\right)$

    $B(t)$: Blaschke factor ($|B(t)|=1$ for $t>t_+$, $B(t_i)=0$)

- None of the standard approaches has complete input:
  - modulus $|F(t)|$ poorly known at low energies
  - phase $\delta(t)$ unknown in the inelastic region $t > t_{in}$
  - the possible zeros in the complex plane are not known

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- Present work: conservative use of the available information $\Rightarrow$ upper and lower bounds on the derivatives of $F(t)$ at $t = 0$

Extremal problem

Find bounds upper and lower bounds on the derivatives of $F(t)$ at $t = 0$ from the input conditions:

- $\arg[F(t + i\epsilon)] = \delta_1^1(t), \qquad 4M_\pi^2 \leq t \leq t_{in}, \quad t_{in} = (M_\omega + M_\pi)^2$
- $\frac{1}{\pi}\int\limits_{t_{in}}^{\infty} \rho(t)|F(t)|^2 dt = I,$ for suitable choices of the weight $\rho(t)$
- $F(0) = 1$
- $F(t_n)$ measured at several points $t_n < 0$
- $|F(t_n)|$ measured at some points in the elastic region $t_n < t_{in}$

The problem can be reduced to the analytic interpolation theory for the Hardy space H^2 of analytic functions in the unit disk.

Solution of the extremal problem

Positivity of a determinant and of its minors:

$$\begin{vmatrix} \bar{I} & \bar{\xi}_1 & \bar{\xi}_2 & \cdots & \bar{\xi}_N \\ \bar{\xi}_1 & \frac{z_1^{2K}}{1-z_1^2} & \frac{(z_1 z_2)^K}{1-z_1 z_2} & \cdots & \frac{(z_1 z_N)^K}{1-z_1 z_N} \\ \bar{\xi}_2 & \frac{(z_1 z_2)^K}{1-z_1 z_2} & \frac{(z_2)^{2K}}{1-z_2^2} & \cdots & \frac{(z_2 z_N)^K}{1-z_2 z_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \bar{\xi}_N & \frac{(z_1 z_N)^K}{1-z_1 z_N} & \frac{(z_2 z_N)^K}{1-z_2 z_N} & \cdots & \frac{z_N^{2K}}{1-z_N^2} \end{vmatrix} \geq 0$$

$$\bar{I} = I - \sum_{k=0}^{K-1} g_k^2, \quad g_k = \left[\frac{1}{k!}\frac{d^k g(z)}{dz^k}\right]_{z=0}, \quad 0 \leq k \leq K-1$$

$$z_n = z_n^*, \qquad \bar{\xi}_n = \xi_n - \sum_{k=0}^{K-1} g_k z_n^k, \quad \xi_n = g(z_n), \quad 1 \leq n \leq N$$

Solution of the extremal problem

$$g(z) \equiv F(\tilde{t}(z,t_0))\,[\mathcal{O}(\tilde{t}(z,t_0))]^{-1}\,w(z)\,\omega(z)$$

- $z \equiv \tilde{z}(t,t_0)$ conformal mapping of the t-plane cut for $t > t_{in}$ onto a unit disk, with the inverse $\tilde{t}(z,t_0)$:

$$\tilde{z}(t,t_0) = \frac{\sqrt{t_{in}-t_0}-\sqrt{t_{in}-t}}{\sqrt{t_{in}-t_0}+\sqrt{t_{in}-t}}, \qquad \tilde{z}(t_0,t_0)=0, \qquad (t_0=0)$$

- $\mathcal{O}(t)$ is the Omnès function:

$$\mathcal{O}(t) = \exp\left(\frac{t}{\pi}\int_{t_+}^{\infty} dt\,\frac{\delta(t')}{t'(t'-t)}\right)$$

- $w(z)$ and $\omega(z)$ are outer functions, *i.e.* analytic and without zeros in $|z|<1$

$$w(z) = \exp\left[\frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta\,\frac{e^{i\theta}+z}{e^{i\theta}-z}\,\ln[\rho(\tilde{t}(e^{i\theta},t_0))|d\tilde{t}/dz|]\right]$$

$$\omega(z) = \exp\left(\frac{\sqrt{t_{in}-\tilde{t}(z,t_0)}}{\pi}\int_{t_{in}}^{\infty}\mathrm{d}t'\,\frac{\ln|\mathcal{O}(t')|}{\sqrt{t'-t_{in}}(t'-\tilde{t}(z,t_0))}\right)$$

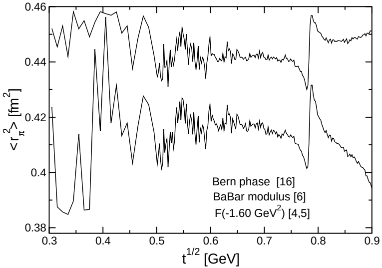
Particular case

- three derivatives at $z = 0$ $(K = 4)$
  - $g(0)$ and $g^k(0)$ expressed in terms of $F(0) = 1$, the charge radius $\langle r_\pi^2 \rangle$, c and d
- $N = 2$ interior points $t_n < t_{\rm in}$
  - a spacelike point $t_1 < 0$
  - a timelike point $t_2 < t_{\rm in}$ where the modulus $|F(t)|$ is measured

$\Rightarrow$ The determinant inequality provides an explicit quadratic equation from which upper and lower bounds on the quantities of interest are obtained

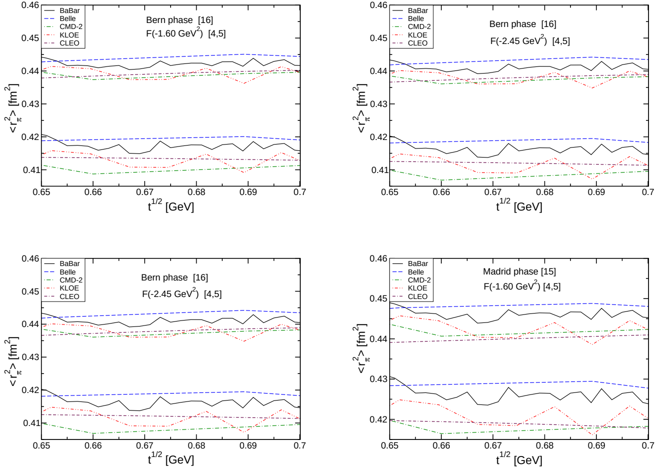
Rigorous properties of the bounds

- are optimal for a given input, *i.e.* imply no loss of information
- are model independent, *i.e.* do not rely on specific parametrizations
- are independent on the arbitrary phase $\delta(t)$ for $t > t_{in}$ Abbas, Caprini et al, EPJA (2010)
- are independent of the conformal mapping (the parameter t_0)
- depend in a monotonous way on I: smaller $I \Rightarrow$ stronger constraints
- the uncertainty of the input quantities can be implemented in a systematic way



- Upper and lower bounds on $\langle r_\pi^2 \rangle$ using as input one modulus value below the $\omega\pi$ inelastic threshold from BaBar experiment as functions of the energy $\sqrt{t}$ where the modulus was implemented
- At each energy the input was varied within the errors and the most conservative bounds were taken
- Big fluctuations with respect to the input modulus. Indicate inconsistencies in the data especially at low energies, in spite of the larger errors

Charge radius analysis



Upper and lower bounds on $\langle r_\pi^2 \rangle$ using as input one modulus value in the region of stability.

Charge radius analysis

- Take the intersections of the ranges of $\langle r_\pi^2 \rangle$ obtained with input modulus at various energies, *i.e.* the smallest upper bound and the largest lower bound
- If the final lower bound ($l.b.$) is greater than the final upper bound ($u.b.$) the intersection is empty. This is the case is all the points are included
- A nonempty intersection is obtained if we consider only the points from the stable region 0.65 – 0.70 GeV

$F(-t_n)$	$\|F(t)\|$	Bern phase				Madrid phase			
		All points included		(0.65 – 0.70) GeV		All points included		(0.65 – 0.70) GeV	
		$l.b.$	$u.b.$	$l.b.$	$u.b.$	$l.b.$	$u.b.$	$l.b.$	$u.b.$
t_1	Belle	0.4229	0.4028	0.4200	0.4428	0.4362	0.4254	0.4294	0.4463
	BaBar	0.4562	0.4299	0.4210	0.4404	0.4455	0.4343	0.4302	0.4435
	CMD2	0.4302	0.4278	0.4125	0.4373	0.4357	0.4288	0.4190	0.4406
	KLOE	0.4264	0.4255	0.4158	0.4362	0.4430	0.4302	0.4248	0.4385
t_2	Belle	0.4227	0.4027	0.4194	0.4418	0.4368	0.4250	0.4301	0.4455
	BaBar	0.4562	0.4279	0.4206	0.4391	0.4455	0.4325	0.4309	0.4425
	CMD2	0.4310	0.4257	0.4109	0.4360	0.4357	0.4285	0.4188	0.4395
	KLOE	0.4267	0.4234	0.4148	0.4348	0.4430	0.4282	0.4250	0.4373

Our prediction based on the intersection:

$$\langle r_\pi^2 \rangle \in (0.42\,,0.44)\,\mathrm{fm}^2$$

Alternative definition: average of the upper and lower bounds

$$\langle r_\pi^2\rangle_{av} = \frac{\sum_n w_n \langle r_\pi^2\rangle_n}{\sum_n w_n}, \quad w_n = 1/\epsilon_n^2, \qquad |F(t_n)| = F_n \pm \epsilon_n, \quad t_+ < t_n < t_{\rm in}$$

Weighted averages of upper and lower bounds for $\langle r^2\rangle$ obtained with various inputs:

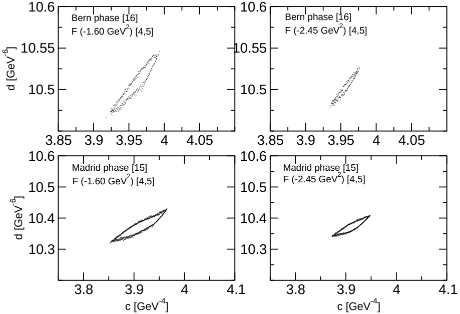
$F(-t_n)$	$\lvert F(t)\rvert$	Bern phase				Madrid phase			
		All points included		(0.65 – 0.70) GeV		All points included		(0.65 – 0.70) GeV	
		$l.b.$	$u.b.$	$l.b.$	$u.b.$	$l.b.$	$u.b.$	$l.b.$	$u.b.$
t_1	Belle	0.4152	0.4443	0.4209	0.4456	0.4187	0.4455	0.4269	0.4475
	BaBar	0.4168	0.4467	0.4194	0.4439	0.4209	0.4474	0.4255	0.4458
	CMD-2	0.4127	0.4443	0.4133	0.4408	0.4122	0.4439	0.4179	0.4421
	KLOE	0.4051	0.4470	0.4151	0.4408	0.4055	0.4453	0.4205	0.4421
t_2	Belle	0.4137	0.4433	0.4204	0.4448	0.4180	0.4446	0.4275	0.4468
	BaBar	0.4155	0.4460	0.4187	0.4430	0.4203	0.4466	0.4259	0.4450
	CMD2	0.4107	0.4432	0.4118	0.4396	0.4102	0.4428	0.4177	0.4411
	KLOE	0.4013	0.4461	0.4139	0.4397	0.4021	0.4442	0.4204	0.4411

Results:

$\langle r_\pi^2\rangle \in (0.41\,, 0.45)\,\mathrm{fm}^2$ using all the data points

$\langle r_\pi^2\rangle \in (0.42\,, 0.44)\,\mathrm{fm}^2$ using only the data from the stable region

For a fixed input, the allowed domain is an ellipse in the c-d plane. Accounting for the errors of the input and taking the intersection give slightly more complicated domains.

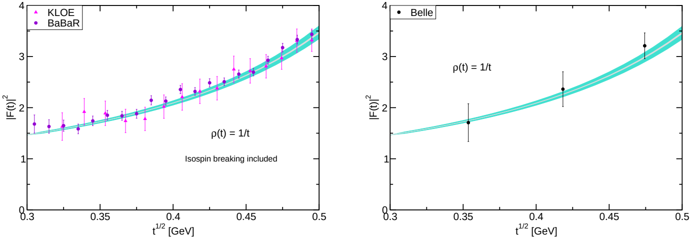


We predict the conservative ranges:

$$c \in (3.86,\, 4.05)\,\mathrm{GeV}^{-4}, \qquad d \in (10.32,\, 10.58)\,\mathrm{GeV}^{-6}$$

with a strong correlation between the parameters.

We can use the formalism in the opposite way and calculate upper and lower bounds on the modulus at low energies, using the input range $\langle r_\pi^2 \rangle \in (0.42\,, 0.44)\,\mathrm{fm}^2$ for the radius



The bounds are more stringent than the present experimental data. Can be used for a more precise determination of muon's anomaly.

Summary and conclusions

- we have studied the impact of the timelike data on the modulus of the pion form factor below the inelastic threshold on the determination of the charge radius $\langle r_\pi^2 \rangle$ and the higher shape parameters c and d in the Taylor expansion at $t = 0$

- the formalism is parametrization-free and does not require ad-hoc assumptions about the form factor at points not accessible to experiment or theory

- the formalism also acts as a sensitive devise for testing the consistency of the various experimental data sets

- the prediction for the charge radius is consistent with most of the results based on specific parametrizations reported in the literature, while for the higher shape parameters the predicted ranges are more stringent than the previous ones

- the analysis illustrates the usefulness of adequate analytic tools in conjunction with high accuracy data for improving the description of the pion electromagnetic form factor

- the results are of interest for testing the expansions of ChPT and the problem of muon's $g - 2$

Relevant publications

1. B. Ananthanarayan, I. Caprini, D. Das and I. Sentitemsu Imsong, *Parametrization-free determination of the shape parameters of the pion electromagnetic form factor*, arXiv:1302.6373 [hep-ph], submitted to EPJC
2. B. Ananthanarayan, I. Caprini, D. Das and I.S. Imsong, *Model independent bounds on the modulus of the pion form factor on the unitarity cut below the $\omega\pi$ threshold*, Eur. Phys. J. **72** (2012) 2192, [arXiv:1209.0379 [hep-ph]].
3. B. Ananthanarayan, I. Caprini and I.S. Imsong, *Spacelike pion form factor from analytic continuation and the onset of perturbative QCD*, Phys. Rev. D **85** (2012) 096006, [arXiv:1203.5398 [hep-ph]].
4. I. Caprini, G. Colangelo and H. Leutwyler, *Regge analysis of the $\pi\pi$ scattering amplitude*, Eur. Phys. J. C **72** (2012) 1860, [arXiv:1111.7160 [hep-ph]].
5. B. Ananthanarayan, I. Caprini and I.S. Imsong, *Implications of the recent high statistics determination of the pion electromagnetic form factor in the timelike region*, Phys. Rev. D **83** (2011) 096002, [arXiv:1102.3299 [hep-ph]].
6. G. Abbas, B. Ananthanarayan, I. Caprini, I. Sentitemsu Imsong and S. Ramanan, *Theory of unitarity bounds and low energy form factors*, Eur. Phys. J. A **45** (2010) 389, [arXiv:1004.4257 [hep-ph]].
7. I. Caprini, *Dispersive and chiral symmetry constraints on the light meson form-factors*, Eur. Phys. J. C **13** (2000) 471 [hep-ph/9907227].